Working my way through Baby Rudin: Every neighborhood is an open set

Marc Schmidt Pujol • 2 Sep 2024

Al right, I'm reading Baby Rudin on my own, and I'm going to share the last proof I've understood. It's in the second chapter: topology.

As usual this proofs in the book comes with the absolute minimum explanation, so here I'm going to add my explanations.

Theorem Every neighborhood is an open set.

'So here the first difficulty for me was to grasp that once we pick a neighborhood E, for any point q inside of this neighborhood we need to show that we can form another neighborhood around q that is fully contained in the first (bigger) neighborhood.'

Proof Consider a neighborhood $E = N_r(p)$, and let q be any point of E. Then there exists a positive real h such that:

$$d(p, q) < r - h.$$

For all points s such that $d(q, s) < h$ *'this is the second neighborhood around q'*, we have then

$$d(p, s) \leq d(p, q) + d(q, s) < r - h + h = r.$$

So that $s \in E$. Thus q is an interior pont of E. ■