Axler, Linear Algebra Done Right, Chapter 1.A Exercises

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Problem 1. Show that $\alpha + \beta = \beta + \alpha$ for all $\alpha, \beta \in \mathbb{C}$.

Proof. Let $\alpha, \beta \in \mathbb{C}$, and write $\alpha = a + bi$ and $\beta = c + di$ for $a, b, c, d \in \mathbb{R}$. We then find that

$$\begin{aligned}\alpha + \beta &= (a + bi) + (c + di) \\ &= (a + c) + (b + d)i \\ &= (c + a) + (d + b)i \\ &= (c + di) + (a + bi) \\ &= \beta + \alpha,\end{aligned}$$

where the third line follows from the fact that addition in $\mathbb{R}$ is commutative.

Problem 2. Show that $(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda)$ for all $\alpha, \beta, \lambda \in \mathbb{C}$.

Solution. Let $\alpha, \beta, \lambda \in \mathbb{C}$, and write $\alpha = a + bi$, $\beta = c + di$, and $\lambda = e + fi$ for $a, b, c, d, e, f \in \mathbb{R}$. One then finds that

$$\begin{aligned}(\alpha + \beta) + \lambda &= ((a + bi) + (c + di)) + (e + fi) \\ &= ((a + c) + (b + d)i) + (e + fi) \\ &= ((a + c) + e) + ((b + d) + f)i \\ &= (a + (c + e)) + (b + (d + f))i \\ &= (a + bi) + ((c + e) + (d + f)i) \\ &= (a + bi) + ((c + di) + (e + fi)) \\ &= \alpha + (\beta + \lambda),\end{aligned}$$

where the fourth equality follows from associativity of addition in $\mathbb{R}$.