How I explain algebraic geometry to non mathematicians

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A circle is a geometric shape we're all familiar with. Many of us have also come across the equation of a circle in high school: $x^2 + y^2 = r^2$, where r is the radius of the circle (we can assume for the rest of the blog for simplicity that the radius is 1). This means that if we were to draw a circle on the two dimensional xy-plane, we would be looking at points of the form (x, y) satisfying the above equation. Therefore, we describe the geometric shape of the circle using an algebraic equation. This is the essence of algebraic geometry.

There's a little caveat here.

If I simply gave you the equation $x^2 + y^2 = 1$ and asked for all the solutions, you may say it's all the points of a unit circle in a 2-dimensional plane. These would indeed give us all the *real* solutions, i.e., points (x, y), where x and y are real numbers, and satisfy $x^2 + y^2 = 1$. However, you may interpret my question as asking for all the *integer* solutions of the equation, in which case there would be only two: $(1, 0)$ and $(0, 1)$. You may also think I'm asking for the *rational* solutions, in which case you'd again have infinitely many solutions, but we would be missing some points from the real circle. For instance, if you pick your favorite irrational number between 0 and 1, say $1/\pi$, then the point $(1/\pi, \sqrt{1 - 1/\pi^2})$ is a real solution but not a rational one. The algebraic equation alone therefore is not sufficient to describe the geometric shape. We also need to emphasize where we want the solutions of the equation to lie. If you know what rings and fields are in abstract algebra, we would call this the base ring/ field (or more generally the base scheme).

Now I could also be asking for solutions coming from the field of complex numbers i.e. numbers of the form $a + ib$, where a and b are real numbers. If z is *any* complex number, then $(z, \sqrt{1 - z^2})$ would be a point in the unit circle over the complex numbers. This is because $\sqrt{1 - z^2}$ is always a meaningful complex number since unlike the reals, every complex number has two meaningful square roots. If we want to understand these points in terms of real numbers, we write $z = a_1 + ib_1$ and $\sqrt{1 - z^2} = a_2 + ib_2$. However, we are now in a 4-dimensional

space since the points (a_1, b_1, a_2, b_2) have 4 coordinates. This is something we unfortunately cannot visualize. But this is the power of algberaic geometry: we can very precisely describe geometric shapes we cannot even visualize using algebraic equations!

But why would we want to be able to do math in higher dimensions that we can't visualize? For many mathematicians, including myself, the fact that we *can* do it is a cool enough reason to *want* to do it. However, this does have some real world applications too: many real world quantities depend on multiple independent factors (often the dependence is given by algebraic equations). Each of these independent factors can be thought of as a dimension. A dimension doesn't have to be a physical direction you can move in. In many areas of physics, time is treated as a fourth dimension.

The solutions to these kinds of algebraic equations no longer correspond to points in physical geometric shapes, but we can still do all the same kinds of algebra with them. For instance, a winter-coat retailer might use a simplified seasonal pricing model in which coats become cheaper as the weather warms. If P is the price in dollars and T is the temperature in Farheneit, the dependence could be

$$\left(\frac{P-50}{150}\right)^2 + \left(\frac{T-14}{54}\right)^2 = 1.$$

This gives a price of $\$200$ at $14°F$ and $\$50$ at $68°F$. These numbers don't describe positions on a physical circle, but after scaling, they follow the same equation as a unit circle. If we treat two pairs as complex numbers and multiply them, the new pair still follows that equation, and could give another value of coat price and the corresponding temperature (just like multiplying two points in a unit circle will give us a third point in the circle). Although of course, in this case, it might give a negative price in which case the model stops making sense!