Seven Sketches in Compositionality - Exercise 2.68

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Problem

Find another monoidal monotone $g : \mathbf{Cost} \to \mathbf{Bool}$ different from the one defined in Eq. 2.66.

Answer

Consider

$$g(x) = \begin{cases} \texttt{true} \text{ if } x < \infty, \\ \texttt{false} \text{ if } x = \infty. \end{cases}$$

We have already shown (Ex. 2.44) that g is a monoidal monotone. Thus, g can turn a Lawvere metric space into a preorder. Intuitively, the preorder relationship will be the one of "reachability": $x \leq y$ if and only if y can be reached from x in a finite time going at a finite velocity.