

# Seven Sketches in Compositionality - Exercise 2.68

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## Problem

Find another monoidal monotone  $g : \mathbf{Cost} \rightarrow \mathbf{Bool}$  different from the one defined in Eq. 2.66.

## Answer

Consider

$$g(x) = \begin{cases} \text{true} & \text{if } x < \infty, \\ \text{false} & \text{if } x = \infty. \end{cases}$$

We have already shown (Ex. 2.44) that  $g$  is a monoidal monotone. Thus,  $g$  can turn a Lawvere metric space into a preorder. Intuitively, the preorder relationship will be the one of "reachability":  $x \leq y$  if and only if  $y$  can be reached from  $x$  in a finite time going at a finite velocity.