Loopy Games

Hypersurreal • 8 Dec 2023

Idempotent	(Nonzero) Loopfree Games Absorbed
$\mathbf{on} = \{\mathbf{pass}\|\}$	All games
$\mathbf{over} = \{0\|\mathbf{pass}\}$	All infinitesimals
$\mathbf{star}_n = \{0\|\|0, *n\|0, \mathbf{pass}\}\ (n \geq 2)$	$*n$ and $\uparrow^2$, but not $*m$ for any $m \neq n$
$\mathscr{I}_n = \{0\|\|0, \mathbf{pass}\|0, \downarrow_{\to(n-2)} *\}\ (n \geq 2)$ $\mathscr{J}_n = \{0\|\|0, \downarrow_{\to(n-1)} *\|0, \mathbf{pass}\}\ (n \geq 2)$	$\uparrow^n$ but not $\uparrow^{n-1}$
$\uparrow^{\mathbf{on}} = \{0\|\|0\|0, \mathbf{pass}\}$	"Almost tiny" all-smalls (such as $\{0\|\|0\|\Downarrow\}$), but not $\uparrow^n$ for any n
$\dotplus_{\mathbf{over}} = \{0\|\|0\|\mathbf{under}\}$	All tinies, but no all-smalls
$\dotplus_{x\mathbf{under}} = \{0\|\|0\|-x\mathbf{over}\}\ (x > 0)$	$\dotplus_{x\downarrow n}$, but not $\dotplus_{x-2^{-n}}$ for any n
$\mathscr{T}_x = \{0\|\|0\|-x, \mathbf{pass}\}\ (x > 0)$	$\dotplus_y$ for all $y > x$, but not $\dotplus_x$
$\dotplus_{x\mathbf{over}} = \{0\|\|0\|-x\mathbf{under}\}\ (x > 0)$	$\dotplus_{x+2^{-n}}$ for all n, but not $\dotplus_{x\uparrow n}$
$\dotplus_{\mathbf{on}} = \{0\|\|0\|\mathbf{off}\}$	None

> One of the most fascinating aspects of loopy games [is] they often witness precise limits for natural sequences of loopfree games.
> -Stoppers as Limits (Siegel)

$$on = \{on|\}$$

$$off = \{|off\}$$

$$tis = \{tisn|\}$$

$$tisn = \{|tis\}$$

$$onto = \{onfro|\}$$

$$onfro = \{onto|\}$$

$$upon = \{upon|*\}$$