

Nimbers

Hypersurreal • 4 Dec 2023

$$\text{nim} \equiv \odot$$

Keep thinking about nimbers..

I recently had MathGPT (a custom version of ChatGPT) help me with a “setup” that seems to work?

We start off with the empty set denoted as 0 (representing \emptyset) & the power set of the empty set denoted as * (representing $P(\emptyset) = \{\emptyset\}$).

Then we have boolean ring operations.

For addition we use xor (\oplus), which equates to the symmetric difference in set theory, such that:

$$0 \oplus 0 = 0$$

$$0 \oplus * = *$$

$$* \oplus * = 0$$

For multiplication we use logical and (\wedge), which equates to the intersection in set theory, such that:

$$0 \wedge 0 = 0$$

$$0 \wedge * = 0$$

$$* \wedge * = *$$

This seems to give us the correct relationships. But I haven't gone much further w/ it..

That wasn't really what was on my mind tho.. for whatever reason I wanted to go thru the nimber construction.

The Surreal number construction starts w/ the empty set & zero:

$$\emptyset = \{ \}$$

$$0 = \{ \emptyset | \emptyset \}$$

It has been shown that the class of impartial games (where both players have the same options) is equivalent to numbers. Since both players have the same options, impartial games are often written w/out the | notation:

$$x = \{y|y\} \implies x = \{y\}$$

When we talk about numbers, we often see 0^* & 1^* which are equivalent to 0 & $*$ respectively.

Each number “contains” each previous number, such that:

$$0^* = \{\}$$

$$1^* = \{0^*\}$$

$$2^* = \{0^*, 1^*\}$$

$$3^* = \{0^*, 1^*, 2^*\}$$

In Winning Ways (Volume 2?) the following formula is given for \odot (sunny) (aka the set of all numbers):

$$\odot = \{0^*, 1^*, 2^*, \dots\}$$

The context is however finite numbers.

In other contexts it is shown that

$$\omega^* = \{0^*, 1^*, 2^*, \dots\}$$

When given the context of transfinite numbers, it seems logical to take \odot as

$$\odot = \{0^*, 1^*, \dots, \omega^*, \dots\}$$

Similar to $\Omega(\equiv \text{on})$ in \mathcal{O}_n .

Another interesting thing about numbers is that for any number n we have $n + n = 0$.

Perhaps trivial equations, but I like the aesthetics:

$$\text{nim} + \text{nim} = 0$$

$$\odot + \odot = 0$$