Nimbers

written by Hypersurreal on Functor Network
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$$\mathbb{nim} \equiv \odot$$

Keep thinking about nimbers..

I recently had MathGPT (a custom version of ChatGPT) help me with a "setup" that seems to work?

We start off with the empty set denoted as 0 (representing $\emptyset$) & the power set of the empty set denoted as $*$ (representing $P(\emptyset) = \{\emptyset\}$).

Then we have boolean ring operations.

For addition we use xor ($\oplus$), which equates to the symmetric difference in set theory, such that:

$$0 \oplus 0 = 0$$

$$0 \oplus * = *$$

$$* \oplus * = 0$$

For multiplication we use logical and ($\wedge$), which equates to the intersection in set theory, such that:

$$0 \wedge 0 = 0$$

$$0 \wedge * = 0$$

$$* \wedge * = *$$

This seems to give us the correct relationships. But I haven't gone much further w/ it..

That wasn't really what was on my mind tho.. for whatever reason I wanted to go thru the nimber construction.

The Surreal number construction starts w/ the empty set & zero:

$$\emptyset = \{\}$$

$$0 = \{\emptyset|\emptyset\}$$

It has been shown that the class of impartial games (where both players have the same options) is equivalent to nimbers. Since both players have the same options, impartial games are often written w/out the | notation:

$$x = \{y|y\} \implies x = \{y\}$$

When we talk about nimbers, we often see $0*$ & $1*$ which are equivalent to 0 & $*$ respectively.

Each nimber "contains" each previous nimber, such that:

$$0* = \{\}$$

$$1* = \{0*\}$$

$$2* = \{0*, 1*\}$$

$$3* = \{0*, 1*, 2*\}$$

In Winning Ways (Volume 2?) the following formula is given for $\circledcirc$ (sunny) (aka the set of all nimbers):

$$\circledcirc = \{0*, 1*, 2*, ..\}$$

The context is however finite nimbers.

In other contexts it is shown that

$$\omega* = \{0*, 1*, 2*, ...\}$$

When given the context of transfinite nimbers, it seems logical to take $\circledcirc$ as

$$\circledcirc = \{0*, 1*, ..., \omega*, ...\}$$

Similar to $\Omega(\equiv \text{on})$ in $\mathbb{O}n$.

Another interesting thing about nimbers is that for any nimber n we have $n + n = 0$.

Perhaps trivial equations, but I like the aesthetics:

$$\mathbb{nim} + \mathbb{nim} = 0$$

$$\circledcirc + \circledcirc = 0$$