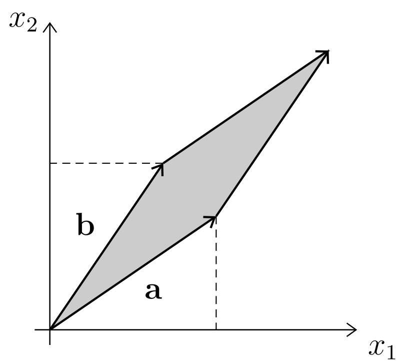
A geometrical explanation of the determinant formula for 2×2 matrices

Words. And some formulas. • 11 Mar 2024

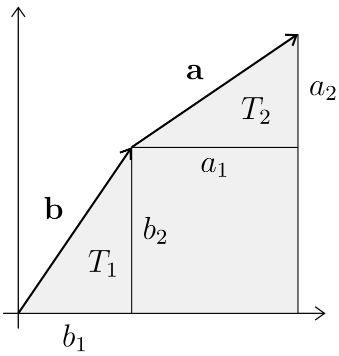
The determinant of the linear function that maps the two-dimensional vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ to $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ is

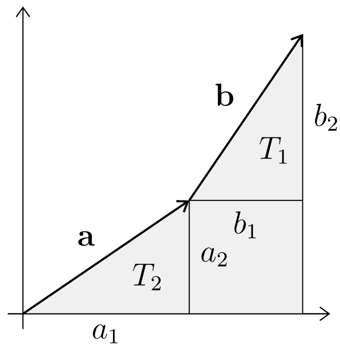
$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1 \,. \qquad (*)$$

It can also be viewed as the area of the parallelogram that is spanned by the vectors $\mathbf{a}$ and $\mathbf{b}$:



With this definition, the formula $(*)$ can easily be proved geometrically. This is because the area of the grey parallelogram is the difference of the two areas

$A_1 =$ 

and $A_2 =$ 

The triangles T_1 and T_2 occur in both diagrams, therefore the value of the determinant is $A_1 - A_2 = a_1 b_2 - b_1 a_2$.