Be ruthless! A list of what makes a math textbook good

Kamil'σ math blog • 5 Mar 2024

TLDR: Don’t waste time on bad textbooks. Be harsh. Look for textbooks with:

- # examples / # definitions ≈ 1.
- Exercises with solutions.
- Jokes, and more!

This is a “crosspost” of the original from my homepage (I update it more often).

Be ruthless! A list of what makes a math textbook good

There are many bad textbooks. They pose a great danger for you. Beware!

I wrote this list for myself. I wrote it with the hope that the next time I will pick a book and think “Oh, this looks nice. Maybe it is okay that there are no examples whatsoever.” I will go here and this list will help me to make a hard decision to throw that book.

Here is a collaborative list of Good Math Textbooks ®. Feel free to add your suggestions and I will look into tem.

A good textbook…

…has exercises with solutions

This is the single and most important thing.

- In *Vector calculus, linear algebra, and differential forms: a unified approach* after every concept there is a quick exercise. The solutions are on the same page, so there is no traction. Perfect! ε>

…explains why and gives context

Why a particular theorem is important or interesting?

- *Analysis I* begins with “Why do analysis?”.
- In *Linear Algebra, Pillar I* the chapter about determinants begins with “Why determinants?”
- In *Precalculus* the reader finds remarks on why or when certain skills will be important later.

…provides many examples, non-examples and counterexamples

- *Seven Sketches on Compositionality* has roughly the same number of definitions and the number of examples. Fire!

…gives glimpses of advanced topics & communicates limitations

- In *Precalculus* authors mention a naive set theory and its fundamental flaw. The authors provide further readings for the curious high schoolers.

…cares about definitions and notation

A reader needs a signal when the notation is overused. Or when a phrase is just a remnant of somebody’s mistake.

- From some uncareful biology textbooks you will learn that “breaking bonds releases energy”. But chemists, correctly, will say the exact opposite!
- In *Analysis I* Terence Tao carefully defines a limit of a function at a point. His notation goes a against the grain, but it is more precise and, for me, wasn’t confusing contrary to the standard one. Here is a MSE question about it.

…is honest with the reader

Thurston in *On proof and progress in mathematics* shares his observation that the formal form of papers and textbooks is sometimes an obstacle. Is the author pompous? Do they mention context or informal reasons for doing such and such moves in the proof? Mathematics is done by people, and, as far as my expireience goes, we need some amount of informality.

Status seeking may prevent authors from being honest. For example, an author might avoid mentioning that some of their chapters are boring but necessary. Why?! Knowing this is a valuable information for the reader who is bored and thinks something is wrong with them.

On a different note, I love when an author doesn't hide their enthusiasm. Knowing that some particular piece of text is the author's special interest brings a more engaging, "dialogue-like" energy to the textbook.

I also like to see a few jokes or easter eggs. I think it's a sign that an author enjoyed writing and wasn't treated like a robot by the publishers.

- In the *Introduction to abstract algebra* we read "If you get bored reading it, you have my sympathy. I was bored typing it."
- In *Linear algebra done right* page 21 is written as "≈7π".
- In *The integrals of Lebesgue, Denjoy Perron, and Henstock* we read a joke combined with an advice:

  > The above proof offers another advantage of a variable δ – the point c was forced to be a tag. We will use this device on several other occasions. The reader should pay careful attention to such "tricks". The next theorem looks at the linearity properties of the Henstock integral.

  Did you noticed it? If not, look at cursive "cc". Another sentence from this book: "This lemma, known as the Vitali Covering Lemma, will probably seem strange at first and it may require several readings to make sense of it. It is necessary to see the lemma in action several times before appreciating it."

…mentions metacognitive aspects

How to think about the subject? What metaphors will help? What are the dangers of a particular metaphor? Will your current knowledge interfere with what you are about to learn?

…has neat graphics & layout

Neat graphics not only mean pretty pictures. The reader should be informed what are limitations of a particular picture.

Personally, I'm a fan of hand drawings in textbooks.

- Some biology textbooks don't care about the scale. This is bad. For a neat example of the opposite see Cell Biology by the Numbers.
- In *Road to Reality* there are plenty of author's drawings. I think that you can learn from those how the author *feels* the topic.

…uses simple language

Comparing used language in the below table should be self-explanatory.

Newton's laws as stated in…	
Dynamics and Relativity	Fundamentals of Physics
Left alone, a particle moves with constant velocity.	In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).
The acceleration (or, more precisely, the rate of change of momentum) of a particle is proportional to the force acting upon it.	The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.
Every action has an equal and opposite reaction.	If two objects interact, the force $\mathbf{F_{12}}$ exerted by object 1 on object 2 is equal in magnitude to and opposite in direction to the force $\mathbf{F_{21}}$ exerted by object 2 on object 1: $\mathbf{F_{12}} = \mathbf{-F_{21}}$

…provides summaries & a big picture for each chapter

- *In Linear Algebra Done Right* every chapter is summarized with a big picture & motivations.
- In each chapter of *Online notes for MAT237: Multivariable Calculus, 2018-9* Robert Jerrard writes a "Basic Skills" section.

List of good textbooks®

Good math books.

Disclaimers

This list is written in a style of being harsh as a counterweight to being too soft on textbooks. I have written it during my undergrad years.

I expect the less experienced a reader is the more relevant all of the points may be. I wonder if they stay relevant for me in the comming years.

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