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Circle Tangency Criterion for Cyclic Quadrilateral

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Abstract

We present a bidirectional characterization of cyclic quadrilaterals based on the tangency condition of two localized subcircles generated by diagonal intersections, orthocenter, and a circumcenter.

1. Introduction

The structural properties of cyclic quadrilaterals form a rich chapter in Euclidean geometry. This paper presents a necessary and sufficient condition for a convex quadrilateral to be cyclic: two constructed circles are tangent to each other at the diagonal intersection point.

2. Notations and Definitions

Let $ABCD$ be a convex quadrilateral whose diagonals AC and BD intersect at Q.

- O is the circumcenter of triangle QCD. - N is the midpoint of QC, and M is the midpoint of QD. - ω_1 is the circle through N, M, O, Q, with center α. - H is the orthocenter of triangle QAB. - G is the foot of the perpendicular from B onto QA. - F is the foot of the perpendicular from A onto QB. - ω_2 is the circle through H, Q, G, F, with center β. - I is the foot of the altitude from Q onto AB. - ω is the circumcircle of triangle QCD.

3. Main Theorem

Theorem. The quadrilateral $ABCD$ is cyclic if and only if the circles ω_1 and ω_2 are tangent to each other at Q.

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