

vulpecular musing 12: a humorous coincidence in enumerative combinatorics

vulpecule • 19 Aug 2026

There are ten magmata of order two up to isomorphism. They are as follows:

- The constant semigroup 0_2
- The left zero band LZ_2
- The right zero band RZ_2
- The semilattice (chain) **2**
- The group $\mathbb{Z}_2$
- A nonassociative right rack R_r
- A nonassociative left rack R_ℓ
- A nonassociative, nonrack right shelf S_r
- A nonassociative, nonrack left shelf S_ℓ
- The NAND/NOR magma. $\ominus$

There are also ten bands of order three up to isomorphism. They are as follows:

- The left zero band LZ_3
- The right zero band RZ_3
- The chain **3**
- The “V” semilattice **V**
- $LZ_2 \uplus 0$
- $RZ_2 \uplus 0$
- A miscellaneous band with two left-absorbing elements, LA
- The opposite of the above, RA
- The left flip-flop monoid, T_ℓ
- The right flip-flop monoid. T_r

To be clear, this is merely a coincidence, or at least there is no compelling fact I can think of underlying both numbers, and there is no particular reason to believe there is one, especially as the former has three left-right pairs and the latter has four.

However, this does not preclude the existence of a “nicest bijection” between the two, and it might be a fun exercise to decide which one that is. We clearly want the bijection to send as many left-right pairs as possible to left-right pairs, so we

have a choice of 24 different “cores” that do this, and for each of these 24 “cores” we have a further 24 “shells” to choose, so we naively have 576 choices. However, we definitely want to exchange the left/right zero bands, so we actually only have 6 choices of “core”, and we definitely want to exchange the chains, so we actually only have 6 choices of “shell”, reducing the choices drastically down to 36.

Further, I think we should probably have the racks sent to the zeroes plus zero, as they’re the ones “closer” to being quasigroups in my opinion, leaving 12 choices. Also, I think $\mathbf{V}$ should be exchanged with 0_2 , as they both “sit between” the right and left zeroes in different senses, leaving only four choices.

I think it is *probably* best to exchange RA, LA with S_r, S_ℓ , as they are the most “miscellaneous” pairs, leaving a final pair of bijections to choose between.

One of these exchanges T_r and $\mathbb{Z}_2$ and the other exchanges it and $\ominus$, and I genuinely have no preference between this pair, as I feared.