

vulpecular musing 11: an observation on associativity conditions in hypermagmata

vulpecule • 18 Aug 2026

A **hypermagma** is a multivalued map from A^2 to A , or equivalently, a map from A^2 to $\mathcal{P}(A)$, or even a magma distributing over both sides of meet and join (such that bottom is absorbing) of a Boolean algebra.

The basic adaptation of associativity in a hypermagma (a hypersemigroup if you will) is to simply say that associativity holds setwise - the set $x \cdot yz$ is identical to the set $xy \cdot z$. In the Boolean algebra implementation this is just ordinary associativity.

However, one might feel like this condition, as straightforwardly analogous as it is, to be nonetheless too weak. A hypermagma is said to be **coordinated** if, for every triple x, y, z , there exists $a \in xy$ and $b \in yz$ such that $az = xb$.

However, it actually unexpectedly turns out that all hypersemigroups are coordinated (PROOF FORTHCOMING).

Coordination is actually strictly weaker than associativity, as evidenced by the hypermagma over a two element set $\{a, b\}$ where the only defined product is $ab = \{a, b\}$.

ACTUALLY THIS ENTIRE THING IS SUSPECT AS I CAN ONLY FIND 23 HYPERSEMIGROUPS OF ORDER THREE WHEN THERE SHOULD BE AT LEAST 24