# vulpeculiar musing 10: a silly observation about structures that can be represented as matrices

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A magma on $A$ is a map from $A^2$ to $A$, so a partial magma (or magmoid) is a *partial* map from $A^2$ to $A$ (or equivalently, a map from $A^2$ to $A \cup 0$ where $0$ is a constant guaranteed not to be in $A$).

One may wish to constrain a partial magma to those examples which are in some sense “close to” being total. For instance, one may impose the existence of a submagma of a certain size, or more strongly that there is a submagma including all defined products. This latter imposition, which is extremely strong, is essentially the same thing as imposing that the defined elements of the Cayley table can be arranged to fill a square.

This notion leads to the obvious weakening of defined elements filling a rectangle, which is still very strong in the sense that the excluded cases are really very trivial. So a weaker notion is that defined elements can be arranged to all lie in the same connected blob of von Neumann neighborhoods within the Cayley table. It is clear that not all partial magmata are edge-connectable in this manner, as, at the very least, any partial magma where the only defined products are squares flouts it. An even weaker notion, then, would be a partial magma where defined elements can be arranged to all lie in the same connected blob of *Moore* neighborhoods. It is not as immediately clear that there exist partial magmata which do not satisfy this, but a minimal example can be given by a three-element partial magma wherein the exhaustive list of defined products is $ab$, $bc$, and $ca$.

Now, there is nothing that strictly reserves this notion of connectivity within Cayley tables to partial magmata - it applies just as well to anything that is represented by an equivalence class of matrices under simultaneous permutation of its rows and columns. For instance, one could consider pointed (total) magmata with connected entries of the distinguished constant, or binary relations with connected included pairs.

I rather seriously doubt these notions have much utility in any of these cases, but it is somewhat interesting that these apparently purely geometric impositions actually have any abstract meaning.