

vulpecular musing 7: let's climb some trees

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Trees, as in connected graphs without cycles, have an interesting property. Given any two nodes, there is a unique path connecting them. Therefore, if we have some map from the set of paths to the set of nodes, we define a magma on a given tree.

What are some nice compatibilities we can prescribe between the tree and its magma? There are two that are obvious to me.

- We say a map from paths to nodes, or by extension a tree magma defined with it, **respects nodes** if a path always evaluates to a node it passes through.
- We say a tree magma **respects paths** if the product of the path $x \rightarrow y$ and $y \rightarrow z$ is equal to the path $x \rightarrow z$, or equationally: $xy \cdot yz = xz$. A tree magma satisfying both compatibilities is said to be **arboreal**.

I think we can connect this to semilattices somehow, but it is not as simple as I thought. Further musing required.