vulpeculiar musing 5: plusquam-minimal varieties

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https://functor.network/user/3486/entry/1924

Prerequisites: the notion of a variety, see musing 1.

A minimal variety is a variety whose only proper subvarieties are trivial. They therefore form the atoms of subvariety lattices and as such are pretty important in UA. A key example of a minimal variety is the variety of semilattices.

But there is a compelling sense in which a minimal variety is not actually minimal.

Let's define the notion of a plusquam-subvariety: the collection of algebra $\mathcal{W}$ is a **plusquam-subvariety** of the variety $\mathcal{V}$ if it consists exactly of the $\mathcal{V}$-reducts of some expansion of $\mathcal{V}$, regardless of whether it is actually a subvariety. Clearly, there exist proper nontrivial plusquam-subvarieties of minimal varieties: one may simply look at the semilattice reducts of distributive lattices, or even of Boolean algebra…

This brings up an obvious question: do there exist **plusquam-minimal varieties**, varieties with no proper nontrivial plusquam-subvarieties?

The intuitive answer is "uh, yeah, no", considering how easy it is to construct plusquam-subvarieties, but it is not outside of the realm of my imagination that some pathological example can be constructed. Perhaps further knowledge will come with further delving into the depths of UA…