

First post

Peter Lane • 11 Jul 2026

For now, I'll focus on *topology*.

Topology is a straightforward and fundamental subject. It starts with the simplest ways of thinking about the world. Consider, I have three items of food: chips, fish and ice-cream. What's the first thing I am likely to do with them? Put them into groups: "fish and chips", and then "ice-cream."

Topology begins by thinking about the kinds of groups we can put items into.

We can be a little more formal: the groups will be surrounded by curly brackets, and the whole set of groups forms its own group. Well, I used the word there – these are "sets" and "subsets".

Consider the set: $\{a, b, c\}$.

There are EIGHT possible subsets of those three items:

$$\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

But actually, it is entirely up to us which set of subsets we choose to write down:

$$\{\{a, b\}, \{c\}\}$$

That is another possible set of subsets of the same three items – replace "a" with "chips" etc, and it is the grouping of food we had before.

Now, how about operations on subsets? Can we join two subsets together (union)? Or find the common elements in two subsets (intersection)?

If we take the set of all possible subsets above, we see that unions and intersections are all again in the set, for example: $\{a, b\} \cup \{c\} = \{a, b, c\}$ and $\{a, b\} \cap \{c\} = \emptyset$. But this is not true for the second set of subsets above.

Where the results of the operations are again in the set of subsets, we say the set is "closed" under the union and intersection operations. And we call such a set of subsets a *topology*. Well, almost – there's one wrinkle.

What is a topology? A topology, τ , is a set of subsets for a set X in which:

1. $\emptyset$ and the whole set X are members of τ ;

2. arbitrary unions of members of τ are also in τ – so the topology is closed under the union operation
3. *finite* intersections of members of τ are also in τ – so the topology is closed under the intersection operation, for finite arguments

So there's that wrinkle - "finite" intersections of members of τ are also in τ , but not necessarily "infinite" intersections. Why is this?

Well, for completeness, there are examples of topologies where the finite bit can be dropped; these are called Alexandroff topologies, and I'm sure I'll talk about these more in the future.

But most topologies we come across must obey the finite restriction, and this is because an important example of a topology in much of mathematics is generated by the set of intervals on the real line: (a, b) , where $a \leq b$. Unions and intersections work reasonably smoothly, but it's possible to consider an infinite sequence of intersections:

$$\bigcap_{n=1}^{\infty} \left(-\frac{1}{n}, \frac{1}{n}\right)$$

That intersection tends to a single point, $\{0\}$, which is no longer an interval. For this reason, a topology is restricted to finite intersections.

So, we now know what a topology is. But where does it come from? If someone gives you a set X , where do you begin thinking about a topology? And this is where the interesting part begins, as the topology for a given set is something we control.

We could go crazy, pick a set of subsets and so assign any topology we want to a set, just making sure it meets the rules, for example:

$$\{\emptyset, \{a, b\}, \{c\}, \{a, b, c\}\}$$

This is a valid topology for the set of three items, $\{a, b, c\}$.

But quite often, there might be some properties of the set which we would like to take advantage of: orders and metrics are two examples of such properties.

If our set X has an order, e.g. the trivial order of the natural numbers: $1 \leq 2 \leq 3$, we could form a topology where subsets include all the larger numbers from a minimum:

$$\{\emptyset, \{1, 2, 3\}, \{2, 3\}, \{3\}\}$$

Notice how the set is closed under intersections and unions. Topologies based on the order of the underlying set X can handle arbitrary intersections – these are the Alexandroff topologies, mentioned above.

Another property is if the set X has a distance metric defined on it, e.g. the real plane, $\mathbb{R}^2$. This is an important property, and the functional analysis tower of metric, norm and inner-product spaces is built upon it. Subsets are then defined by how close items are to each other.

Finally, it is worth mentioning that if X is a vector space, we can build topologies to specifically capture the continuity of addition and scalar multiplication in the vector space.