First post

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As my first blog post, I wanted to write about a question I once asked a mentor/ boss of mine last year. He didn't give me the solution, he just said "solve it yourself, give it to me by tonight." I hope before you read the answer, you attempt to solve it yourself first.

Question

You have a portfolio of n different assets. Let $R \in \mathbb{R}^n$ be the vector of returns of all assets. Let $\mu = \mathbb{E}[R]$ be the expectation of the returns, (where returns are excess returns above risk free rate). We want to compose a portfolio that maximizes the expected Sharpe Ratio, defined by:

$$\text{Sharpe Ratio} = \frac{\text{return}}{\text{volatility}}$$

Now the returns depend on how we weight each asset, which is really at the core of the question. Let $w \in \mathbb{R}^n$ be the weight vector, and we enforce these sum to 1, or $\mathbf{1^T w = 1}$. So, what is the optimal weighting scheme to maximize the Sharpe Ratio of the portfolio?

Solution

First, we want to figure out what the sharpe ratio of the portfolio is in terms of the formulas:

$$\text{Portfolio Sharpe Ratio} = \frac{\mathbb{E}[w^T R]}{\sqrt{\text{Var}[w^T\text{R}]}} = \frac{w^T \mu}{\sqrt{w^T \text{Var}[R] w}} = \frac{w^T \mu}{\sqrt{w^T \Sigma w}}$$

Here note that $\Sigma_{i,j} = \text{Cov}(R_i, R_j)$. Now our question formally becomes:

$$\max_{\mathbf{1}^T w = 1} \frac{w^T \mu}{\sqrt{w^T \Sigma w}}$$

From here, one needs to realize that Sharpe Ratio, which is here a function of weight (SR(w)), has the following property

$$\forall c \geq 0, SR(cw) = \frac{c w^T \mu}{\sqrt{c^2 w^T \Sigma w}} = \frac{w^T \mu}{\sqrt{w^T \Sigma w}} = SR(w)$$

This tell us that the sharpe ratio is unchanged by a positive scalar factor of w. Thus we can focus on the direction of w, and later normalize it to satisfy $\mathbf{1}^T w = 1$. So, we can let $w^T\mu = 1$ and then our problem becomes:

$$\max_{w^T\mu=1} \frac{1}{\sqrt{w^T\Sigma w}} = \min_{w^T\mu=1} \sqrt{w^T\Sigma w}$$

So we see minimizing $w^T\Sigma w$ minimizes the function above, thus we can find w by doing:

$$\min_{w^T\mu=1} w^T\Sigma w$$

Now we can solve via Lagrange Multipliers:

$$L(w, \lambda) = w^T\Sigma w - \lambda(1 - w^T\mu) \implies$$

$$\nabla_w L(w, \lambda) = \nabla_w(w^T\Sigma w) - \lambda\nabla_w(1 - w^T\mu) \implies$$

$$\nabla_w L(w, \lambda) = 2\Sigma w + \lambda\mu$$

Setting this equal to zero, we get that:

$$w^* = -\frac{\lambda}{2}\Sigma^{-1}\mu$$

Now we need λ such that $w^T\mathbf{1} = 1$ so we plug in to solve:

$$-\frac{\lambda}{2}(\Sigma^{-1}\mu)^T\mathbf{1} = 1 \implies$$

$$\lambda = \frac{-2}{\mu^T(\Sigma^{-1})^T\mathbf{1}} = \frac{-2}{\mu^T\Sigma^{-1}\mathbf{1}}$$

Finally, putting this together we get that:

$$w^* = \frac{\Sigma^{-1}\mu}{\mu^T\Sigma^{-1}\mathbf{1}}$$

This is the optimal weighting vector to maximize the Sharpe Ratio. Now if we want the max Sharpe itself, we can simply substitute in our weight to the formula:

$$\text{Sharpe}_{w^*} = \frac{\mu^T\Sigma^{-1}\mu}{\sqrt{\mu^T\Sigma^{-1}\mu}} = \sqrt{\mu^T\Sigma^{-1}\mu}$$