Philip Easo's talk on supercritical sharpness of percolation

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This post continues our series of online seminars devoted to accessible presentations of some of the most significant mathematical theorems of the twenty-first century.

I am delighted to report that on November 4, 2026, Prof. Van Vu delivered a lecture on *Current directions in random matrix theory*. If you missed the talk, the recording is now available on YouTube.

The next lecture in the series will take place on **Wednesday, December 2, 2026, at 4 PM (UK time)**. Prof. Philip Easo (University of Geneva, Switzerland) will speak on the topic *Supercritical sharpness of percolation*.

Prof. Philip Easo is a mathematician working in probability theory, particularly on percolation and random processes on graphs. In March 2026, together with Sahar Diskin, Ritvik Ramanan Radhakrishnan, Benny Sudakov, and Vincent Tassion, he posted a preprint (Diskin et al. 2026) proving a major theorem about percolation on infinite transitive graphs. The talk will explain this breakthrough, its background, and related recent developments.

To join the talk, click here at the scheduled time. No registration is required.

For announcements of other talks, as well as descriptions of recent major mathematical breakthroughs, see the full list of posts on this blog.

Let me briefly explain the mathematics of the talk.

Imagine a large network in which every connection independently works with probability p and fails with probability $1-p$. This is the classical model of *Bernoulli percolation*. We are interested in the connected components, or *clusters*, formed by the working connections.

On an infinite transitive graph, such as the square lattice $\mathbb{Z}^2$, there is a critical probability p_c separating two qualitatively different regimes.

When $p < p_c$, every cluster is finite, and the probability that a given vertex belongs to a cluster containing at least n vertices decreases exponentially fast in n.

When $p > p_c$, an infinite cluster can appear. However, a given vertex may still belong to a finite cluster. How likely is it that this finite cluster is very large?

Surprisingly, the answer depends on the geometry of the underlying graph.

For example, on the square lattice $\mathbb{Z}^2$, a connected region containing approximately n vertices can have a boundary consisting of only about $\sqrt{n}$ edges. Thus, to isolate a large cluster from the infinite network, it may be sufficient to close a number of edges proportional to $\sqrt{n}$, rather than n.

This suggests that the probability of a large finite cluster in the supercritical regime should decay roughly like

$$\exp(-c\sqrt{n})$$

for some positive constant c, rather than exponentially in n.

More generally, the relevant quantity is the *isoperimetric function* $\Phi(n)$ of the graph, which measures how small the edge boundary of a finite vertex set containing at least n vertices can be.

In their 2026 paper (Diskin et al. 2026), Diskin, Easo, Ramanan Radhakrishnan, Sudakov, and Tassion proved that, for every infinite connected locally finite transitive graph and every $p > p_c$, the probability that a given vertex belongs to a finite cluster containing at least n vertices is bounded above and below by expressions of the form

$$\exp(-C\Phi(n)) \quad \text{and} \quad \exp(-c\Phi(n)),$$

with suitable positive constants c and C.

Thus, the isoperimetric function determines the decay rate of the probability, up to constant factors in the exponent.

This theorem had previously been known only for special classes of graphs. The new result establishes it for *every* infinite transitive graph, regardless of its geometric structure.

The proof uses an ingenious technique called *sprinkling*: first construct percolation with a slightly smaller parameter, and then independently open some additional edges. The main challenge is to identify sufficiently many opportunities for a large finite cluster to connect to infinity. If the cluster remains finite after sprinkling, all these opportunities must fail, which becomes exponentially unlikely.

I discussed the theorem, its history, and the proof ideas in more detail in my earlier blog post on supercritical sharpness of percolation.

The forthcoming seminar offers an opportunity to hear about this breakthrough directly from one of its authors and to ask questions about the ideas behind the proof.

References

Diskin, Sahar, Philip Easo, Ritvik Ramanan Radhakrishnan, Benny Sudakov, and Vincent Tassion. 2026. *Supercritical Sharpness of Percolation*. https://arxiv.org/abs/2603.03257.