A proof of Sendov's conjecture

Bogdan Grechuk • 17 Aug 2026

On August 5, 2026, Lech Mazur released the manuscript *A Computer-Assisted Proof of Sendov's Conjecture* (Mazur 2026). It presents an exact computer-assisted argument and is accompanied by a separate Lean 4 formalization of the theorem. Mazur is the sole named manuscript author, and the paper discloses substantial assistance from OpenAI's GPT-5.6 Pro in mathematical exploration, proof development, exact testing, auditing, and exposition. On August 12, Terence Tao published a streamlined "digestion" of the argument and a second, substantially shorter Lean formalization (Tao 2026). At the time of writing, ProofAtlas describes Mazur's manuscript as an author-authorized early release for which independent presentation review is still pending (Mazur 2026).

Let p be a complex polynomial. A critical point of p is a zero of its derivative p'. The Gauss–Lucas theorem states that every critical point lies in the convex hull of the zeros of p. In particular, if all zeros of p lie in the closed unit disk

$$\overline{\mathbb{D}} = \{z \in \mathbb{C} : |z| \leq 1\},$$

then all critical points lie there as well. Sendov's conjecture asks for a much more local conclusion: every individual zero must have a critical point within distance one.

The conjecture was introduced by Blagovest Sendov in 1958, although for many years it was attributed to Ilieff; see Marden's survey (Marden 1983). It was proved for polynomials of degree at most eight by Brown and Xiang (Brown and Xiang 1999). Tao's paper, published in 2022, proved it for all sufficiently large degrees (Tao 2022). His argument contained qualitative steps, notably analytic continuation, and did not give a convenient explicit threshold. Thus an intermediate range of degrees remained.

The new result is the following.

Theorem 1 (Sendov's conjecture) *Let p be a complex polynomial of degree $n \geq 2$, all of whose zeros lie in $\overline{\mathbb{D}}$. For every zero a of p, there is a zero w of p' such that*

$$|w - a| \leq 1.$$

The constant 1 is best possible. For

$$p(z) = z^n - 1$$

we have $p'(z) = nz^{n-1}$, so the only critical point is 0, while every zero of p has distance exactly 1 from it. In fact, Tao's digestion shows that the new argument proves the stronger Phelps–Rodriguez conjecture: one can replace ≤ 1 by < 1, unless a lies on the unit circle and p is a scalar multiple of $z^n - a^n$ (Tao 2026).

See Tao's blog post for the proof exposition. The proof is strikingly elementary for a problem about the geometry of complex polynomials. Beyond the fundamental theorem of algebra and a basic Möbius transformation, its main ingredients are coefficient comparisons, integration, the arithmetic mean–geometric mean and Cauchy–Schwarz inequalities, Maclaurin's inequality, and exact finite verification. The central conceptual step is the collapse from two configurations of $n-1$ complex points to two real statistics. This resolves a conjecture open since 1958 and, at the same time, proves its sharp Phelps–Rodriguez strengthening.

References

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