Erdős' sum–product conjecture is false for real numbers

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In a recent preprint (Bloom et al. 2026), Bloom, Sawin, Schildkraut, and Zhelezov proved that the celebrated Erdős–Szemerédi sum–product conjecture is false over the real numbers.

We discussed this conjecture previously in an earlier blog post. For sets $A, B \subset \mathbb{R}$, define

$$A + B = \{a + b : a \in A,\ b \in B\}, \qquad A \times B = \{ab : a \in A,\ b \in B\}.$$

For a positive integer k, define the k-fold sumset and product set by

$$kA = \underbrace{A + \cdots + A}_{k \text{ times}}, \qquad A^k = \underbrace{A \times \cdots \times A}_{k \text{ times}}.$$

Erdős and Szemerédi (Erdős and Szemerédi 1983) conjectured that for every $\epsilon > 0$ there exists a constant $c_\epsilon > 0$ such that, whenever A is a finite set of integers,

$$\max\left(|kA|, |A^k|\right) \geq c_\epsilon |A|^{k-\epsilon}. \tag{1}$$

In other words, repeated addition or repeated multiplication should force nearly maximal growth.

Bloom, Sawin, Schildkraut, and Zhelezov (Bloom et al. 2026) proved that if A is allowed to be a set of arbitrary real numbers, then this conjecture is already false for $k = 2$. More precisely, there is a constant $c > 0$ such that for every N there exists a finite set $A \subset \mathbb{R}$ with $|A| > N$ and

$$\max\left(|A + A|, |A \times A|\right) \leq |A|^{2-c}. \tag{2}$$

The authors also proved a stronger statement for higher-fold sums and products: there exists a constant $C > 0$ such that, for every $k \geq 3$ and every $N > 0$, there exists a finite set $A \subset \mathbb{R}$ with $|A| > N$ and

$$\max\left(|kA|, |A^k|\right) \leq |A|^{C \frac{\log k}{\log \log k}}. \tag{3}$$

Thus, for large k, inequality (1) fails over the real numbers by a very wide margin.

The proof is entirely due to the authors, but it was inspired by an earlier counterexample to the Erdős unit distance conjecture constructed with the help of AI, which we discussed in another previous blog post.

Many attempts to prove (1) have relied on combinatorial methods that apply equally well to real numbers and to integers. We now know that such methods cannot prove the integer conjecture. Any successful proof of (1) must use arithmetic properties of the integers that are not shared by arbitrary real numbers.

References

Bloom, Thomas F, Will Sawin, Carl Schildkraut, and Dmitrii Zhelezov. 2026. “The Sum-Product Conjecture Is False for Real Numbers.” *arXiv Preprint arXiv:2605.28781*.

Erdős, Paul, and Endre Szemerédi. 1983. “On Sums and Products of Integers.” In *Studies in Pure Mathematics*. Springer.