Andrew Lobb’s and Joshua Greene’s talk on Inscription Problems

Bogdan Grechuk • 20 May 2026

This post continues our series of online seminars devoted to accessible presentations of some of the most significant mathematical theorems of the 21st century. I am delighted to report that today, 20 May 2026, Prof. Ben Green delivered an accessible lecture on the polynomial Freiman–Ruzsa conjecture. If you missed the talk, the recording is now available on YouTube.

The next lecture in the series will take place on

Wednesday, 10 June 2026, at 4:00 PM UK time.

Prof. Andrew Lobb (Durham University, UK) and **Prof. Joshua Greene** (Boston College, USA) will speak about inscription problems and symplectic geometry.

To join the talk, simply click here at the scheduled time; no registration is required. For a full list of upcoming seminars, please visit https://th21.le.ac.uk/next-talks/.

For announcements of other talks, as well as descriptions of recent major mathematical breakthroughs, see the full list of posts on this blog.

Below is my description of the topic of the talk. This is **not** the abstract received from the speakers.

A *Jordan curve* is a simple closed curve in the plane, or, more formally, the image Γ of an injective continuous map

$$\gamma : \mathbb{S}^1 \to \mathbb{R}^2 \tag{1}$$

from a circle $\mathbb{S}^1$ into the plane $\mathbb{R}^2$. The fundamental *Jordan curve theorem* states that the complement $\mathbb{R}^2 \setminus \Gamma$ of every such curve Γ consists of exactly two connected components, Ω^+ and Ω^-. Exactly one of these components is bounded, and the curve itself is the common boundary of the two components:

$$\Gamma = \partial\Omega^+ = \partial\Omega^-.$$

A famous family of open problems asks which shapes can be inscribed in an arbitrary Jordan curve. For example, it is known, by a theorem of Myerson (Meyerson 1980), that for any triangle T, every Jordan curve Γ in the plane contains the vertices of a triangle similar to T. The corresponding problem for quadrilaterals is much more difficult. In particular, Toeplitz asked in 1911 whether every plane Jordan curve contains all four vertices of some square. This question, known as the *square peg problem*, remains open. Before 2026, the best progress for general Jordan curves was made by Vaughan, who proved in 1981 that every Jordan curve contains the vertices of some rectangle; see Myerson (Meyerson 1981).

The problem becomes easier for smooth curves, meaning curves for which the function γ in (1) is continuously differentiable. For such curves, the square peg problem was solved positively by Emch (Emch 1916) in 1916. A more general question, the *rectangular peg problem* for smooth curves, asks whether every smooth Jordan curve contains the vertices of a rectangle similar to any prescribed rectangle. Up to similarity, a rectangle is fully determined by its aspect ratio, namely the ratio of its longer side to its shorter side. Before 2018, this problem was open for every rectangle other than the square. In 2018, Hugelmeyer (Hugelmeyer 2018) proved that every smooth Jordan curve has an inscribed rectangle with aspect ratio $\sqrt{3}$. In 2021, he proved the stronger result that every smooth Jordan curve has inscribed rectangles of at least one third of all possible aspect ratios (Hugelmeyer 2021).

Theorem 1 *Let Γ be a smooth Jordan curve in the plane, and let X be the set of all $r \in [0, 1]$ such that there is an inscribed rectangle in Γ of aspect ratio*

$$\tan(r\pi/4).$$

Then the Lebesgue measure of X is at least $1/3$.

Also in 2021, Greene and Lobb (Greene and Lobb 2021) extended Hugelmeyer's ideas and gave a complete solution to the rectangular peg problem for smooth Jordan curves.

Theorem 2 *For every smooth Jordan curve Γ and every rectangle R in the Euclidean plane, there exists a rectangle similar to R whose vertices lie on Γ.*

In 2023, Greene and Lobb (Greene and Lobb 2023) proved the even more general result that Theorem 2 remains true with any cyclic quadrilateral in place of a rectangle. Recall that a quadrilateral is cyclic if it can be inscribed in a circle.

Theorem 3 *For every smooth Jordan curve Γ and every cyclic quadrilateral Q in the Euclidean plane, there exists a quadrilateral similar to Q whose vertices lie on Γ.*

By considering the case in which the Jordan curve Γ is itself a circle, one can easily see that this result is best possible: it cannot hold, in this form, for non-cyclic quadrilaterals. Moreover, the smoothness assumption on Γ cannot simply be omitted from Theorem 3. Indeed, Pak (Pak 2008) proved that the only cyclic quadrilaterals that can be inscribed in every triangle are the isosceles trapezoids. This motivates the *trapezoidal peg problem*, which asks whether every plane Jordan curve inscribes a trapezoid similar to T, for every isosceles trapezoid T. The rectangular peg problem and the square peg problem are special cases, corresponding respectively to rectangles and squares.

In 2024, Asano and Ike posted a preprint (Asano and Ike 2024) in which they resolved the rectangular peg problem for every Jordan curve of finite length. More precisely, if $\gamma : [a, b] \to \mathbb{R}^2$ is a continuous map with image Γ, and if

$$P : \quad a = t_0 < t_1 < \cdots < t_m = b$$

is a partition of $[a, b]$, then the polygonal length associated to P is

$$l(\gamma, P) := \sum_{i=1}^{m} \|\gamma(t_i) - \gamma(t_{i-1})\|.$$

The length of Γ is defined by

$$l(\Gamma) = \sup_{P} l(\gamma, P), \tag{2}$$

where the supremum is taken over all partitions P of $[a, b]$. Asano and Ike proved that if Γ is a Jordan curve with $l(\Gamma) < \infty$, then Γ inscribes a rectangle similar to any prescribed rectangle R. Since every smooth, and more generally every piecewise smooth, Jordan curve has finite length, this is a far-reaching generalization of Theorem 2. However, it does not settle the rectangular peg problem in full generality, because there exist Jordan curves of infinite length.

In 2026, Greene and Lobb posted a preprint (Greene and Lobb 2026) in which they proved that for an arbitrary Jordan curve Γ there is an interval $I_\Gamma \subset (0, \pi)$ of positive measure $\epsilon_\Gamma > 0$ such that, for every $\theta \in I_\Gamma$, the curve Γ inscribes a rectangle whose diagonals meet at angle θ. Moreover, one may take

$$\epsilon_\Gamma = \frac{A(\Gamma)}{R(\Gamma)^2},$$

where $A(\Gamma)$ is the area enclosed by Γ, and $R(\Gamma)$ is half the diameter of Γ.

References

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