Viterbo’s volume-capacity conjecture is false

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The March 2026 issue of the Annals of Mathematics contains a striking paper by Haim-Kislev and Ostrover (Haim-Kislev and Ostrover 2026) which disproves Viterbo’s volume-capacity conjecture, long regarded as one of the central open problems in symplectic geometry.

Symplectic geometry is the study of even-dimensional spaces equipped with a structure that measures oriented two-dimensional areas. To any open region $S \subset \mathbb{R}^2$, we assign an orientation, that is, a choice of direction in which to traverse its boundary ∂S. The *symplectic area* $\omega(S)$ is then defined as a signed quantity: its absolute value $|\omega(S)|$ equals the usual Euclidean area of S, and its sign is positive if the orientation is anticlockwise and negative if it is clockwise.

This notion extends naturally to higher dimensions. For a two-dimensional surface $S \subset \mathbb{R}^{2n}$ equipped with an orientation, its symplectic area is defined by

$$\omega(S) = \sum_{i=1}^{n} \omega(S_i),$$

where S_i denotes the projection of S onto the (x_{2i-1}, x_{2i})-plane. A map $\phi : U \to V$ between subsets of $\mathbb{R}^{2n}$ is called a *symplectic embedding* if it is a diffeomorphism (i.e. a smooth bijection with smooth inverse) that preserves symplectic area, meaning that $\omega(S) = \omega(\phi(S))$ for every surface $S \subset U$.

A fundamental concept in symplectic geometry is that of a *symplectic capacity*. This is a function c assigning to each subset $U \subset \mathbb{R}^{2n}$ a value $c(U) \in [0, \infty]$, subject to the following axioms:

1. *Monotonicity:* if U symplectically embeds into V, then $c(U) \leq c(V)$
2. *Conformality:* for all $\alpha \geq 0$, one has $c(\alpha U) = \alpha^2 c(U)$;
3. *Normalization:* $c(B^{2n}(1)) = c(Z^{2n}(1)) = \pi$, where
   $B^{2n}(1) = \{x \in \mathbb{R}^{2n} : \sum_{i=1}^{2n} x_i^2 \leq 1\}$ is the unit ball, and
   $Z^{2n}(1) = \{x \in \mathbb{R}^{2n} : x_1^2 + x_2^2 \leq 1\}$ is the unit cylinder.

These axioms do not determine c uniquely, and indeed many distinct symplectic capacities have been constructed. Nevertheless, it has been observed that a wide class of such capacities agree on convex domains. This led to a central

conjectural picture: that all symplectic capacities coincide on convex subsets of $\mathbb{R}^{2n}$. Such a result would yield a canonical notion of "symplectic size" for convex bodies and would have far-reaching consequences.

One of these consequences is Viterbo's volume-capacity conjecture (Viterbo 2000), which asserts that for any symplectic capacity c and any convex domain $U \subset \mathbb{R}^{2n}$,

$$c^n(U) \leq n!\, V(U),$$

where $V(U)$ denotes the standard Euclidean volume. Since equality holds for Euclidean balls, the conjecture predicts that among all convex bodies of fixed volume, the ball maximizes symplectic capacity. This statement has been verified in several special cases and, more generally, up to a universal multiplicative constant (Artstein-Avidan et al. 2008).

The recent work of Haim-Kislev and Ostrover overturns this long-standing expectation.

Theorem 1 *Viterbo's volume-capacity conjecture is false: for every $n \geq 2$, there exist a convex domain $U \subset \mathbb{R}^{2n}$ and a symplectic capacity c such that*

$$c^n(U) > n!\, V(U).$$

In particular, there exist convex domains U and symplectic capacities c_1, c_2 such that $c_1(U) \neq c_2(U)$.

This result demonstrates that the landscape of symplectic capacities is fundamentally richer and more intricate than previously believed. It shows that convexity alone does not enforce rigidity among capacities and that the search for a canonical symplectic invariant must take a more nuanced path. More broadly, it serves as a reminder that even well-supported conjectures, connected to multiple strands of mathematical intuition, may ultimately give way to unexpected counterexamples, opening new directions and questions for future research.

References

Artstein-Avidan, Shiri, Vitali Milman, and Yaron Ostrover. 2008. "The M-Ellipsoid, Symplectic Capacities and Volume." *Commentarii Mathematici Helvetici* 83 (2): 359–69.

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