Diameter of Cayley graph of a finite group on the p-singular elements for some prime p

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Let G be a finite group and p be a prime dividing $|G|$. Let $S_p(G)$ be the set of p-singular elements of G i.e., $S_p(G) := \{x \in G \ : \ p \mid o(x)\}$. Here is a conjecture of min.

Conjecture. If G is a solavble group then $\langle S_p(G) \rangle = S_p(G) \cup S_p(G)^2$.

Here, as usual, $X^2 := \{x_1 x_2 \ : \ x_1, x_2 \in X\}$ for a non-empty susbet X of a group.

It is not hard to prove if G is p-nilpotent group then the conjecture is valid.

I have checked for solvable groups of order 96, 1000 and 2000 that the conjecture is true.

To study a minimal counterexample, one may assume that $O_{p'}(G) = 1$.

I have also proposed the conjecture in MatheOverflow

https://mathoverflow.net/questions/504343/diameter-of-p-singular-cayley-graph-of-a-finite-group

I am ready to collaborate on my conjecture if someone interested in. One may reach me via alireza_abdollahi@yahoo.com