Intermediate Counting and Probability - Chapter 1

crashing and burning • 24 Oct 2025

Chapter 1

1.1 This problem is split into two parts.

(a) There are 25 options for the first ball. Since the first ball cannot be the second ball, there are 24 options for the second ball. Similarly, there are 23 options for the third ball and 22 options for the fourth and final ball. So, our final answer is $25 \times 24 \times 23 \times 22 = 303600$.

(b) In this case, there are 25 options for each ball, as there are no restrictions on whether a ball can or cannot be a previous ball. So, our final answer is $25^4 = 390625$.

1.2 In this problem, we do casework on the number of letters in a valid word in the Mumblian language.

Case 1: 1 letter words.
Each letter in these type of words can be any of 5 letters, so this case has $5^1 = 5$ valid words.

Case 2: 2 letter words.
Similarly, each letter in these type of words can be any of 5 letters, so this case has $5^2 = 25$ valid words.

Case 3: 3 letter words.
Finally, in the same way as the last two cases, we get that this case has $5^3 = 125$ valid words.

Now, summing up these cases, we get our answer of $5 + 25 + 125 = 155$ words.

1.3 In this problem, we use complementary counting. That is, we count the number of ways we can arrange the 4 boys and 3 girls in 7 seats such that no two boys are next to each other. It's fairly easy to see that the only arrangement satisfying this is

$$BGBGBGB.$$

Since there are $4! = 24$ ways of arranging the boys and $3! = 6$ ways of arranging the girls, there are $24 \times 6 = 144$ ways of arranging the boys and girls in the ways we don't want.

Now, we count the total number of ways to arrange the 7 people. This is $7! = 5040$. Therefore, our answer is $5040 - 144 = 4896$.

1.4 First, we choose where the 0 goes in the three digit number. There are two slots; the tens digit and the ones digit. Next, we fill in the hundreds digit. This slot can be anything from 1 to 9, so there are 9 ways to fill in this digit. Finally, we fill in the last digit. This digit cannot be 0, but can be anything else, so we have 9 possibilities. Multiplying these, we get our answer of $9 \times 9 \times 2 = 162$.

1.5 We approach this problem via complementary counting. That is, we count the number of ways we can fill in the offices if Ali must serve as an officer if Brenda serves as an officer, and one of them must serve. (Note that if neither of them serve, this still satisfies our initial condition, so we do not count it when complementary counting.)

We have that there are 3 ways to choose which office is not filled by Ali and Brenda, and 2 ways to arrange Ali and Brenda over their two offices. Finally, there are 18 ways to fill in the last office, so our total