Demonstration of the Collatz Conjecture”. Why the Collatz conjecture always ends in the 4, 2, 1 cycle. Analysis of odd numbers and convergence to the 4, 2, 1 cycle.

Gilberto Augusto Carcamo Ortega • 17 Aug 2025

title: “Demonstration of the Collatz Conjecture”. Why the Collatz conjecture always ends in the 4, 2, 1 cycle. Analysis of odd numbers and convergence to the 4, 2, 1 cycle.

author: Gilberto Augusto Carcamo Ortega

To analyze the Collatz conjecture, we will distribute the positive integers into triplets.

k	3k+1	3k+2	3k+3
0	1	2	3
1	4	5	6
2	7	8	9
3	10	11	12
4	13	14	15
5	16	17	18
6	19	20	21
7	22	23	24
8	25	26	27
9	28	29	30

k	3k+1	3k+2	3k+3
10	31	32	33
11	34	35	36
12	37	38	39

From that table, the following can be observed: * If the index k is even, the triplets are {Odd, Even, Odd}. * If the index k is odd, the triplets are {Even, Odd, Even}.

We could represent it in the following way:

k	3k+1	3k+2	3k+3
0	n(mod2)=1	n(mod2)=0	n(mod2)=1
1	n(mod2)=0	n(mod2)=1	n(mod2)=0
2	n(mod2)=1	n(mod2)=0	n(mod2)=1
3	n(mod2)=0	n(mod2)=1	n(mod2)=0
4	n(mod2)=1	n(mod2)=0	n(mod2)=1
5	n(mod2)=0	n(mod2)=1	n(mod2)=0
6	n(mod2)=1	n(mod2)=0	n(mod2)=1
7	n(mod2)=0	n(mod2)=1	n(mod2)=0
8	n(mod2)=1	n(mod2)=0	n(mod2)=1
9	n(mod2)=0	n(mod2)=1	n(mod2)=0
10	n(mod2)=1	n(mod2)=0	n(mod2)=1
11	n(mod2)=0	n(mod2)=1	n(mod2)=0
12	n(mod2)=1	n(mod2)=0	n(mod2)=1

The Collatz conjecture has certain interesting aspects, among them the fact of multiplying every odd number by 3 and then adding 1 (3k+1), we will focus our analysis on this.

To do this, we will define some basic rules regarding the analysis of triplets that will help us understand the Collatz conjecture a little more.

Mathematics for the analysis of triplets

- If we select an odd index k, the result of 3k+1 will be an even number.
- If we divide an even number obtained from the 3k+1 operation (for a given index k) by two, the result will be a number of the form 3j+2, where j is the quotient of dividing k/2. The operation can be written as: j = floor(k/2).
- If we divide an even number from the 3k+2 operation (for a given index k) by two, the result will be a number of the form 3j+1, where j is the quotient of dividing k/2. The operation can be written as: j = floor(k/2).
- Every k-index of the form k=1+4(n-1) is an odd number that within the Collatz conjecture will generate an even number of the form ck=4+12(n-1), which will be divisible by four. If the number is divisible by two an even number of times, the number will converge upward; if, on the contrary, it is divisible an odd number of times, it will not converge upward in the first interactions of the Collatz rules.
- Every k-index of the form k=3+4(n-1) is an odd number that within the Collatz conjecture will generate an even number of the form ck=10+12(n-1), which will be divisible by two only once. This type of number does not converge upward in the first iterations of the Collatz rules.
- If the k-index is of the form k=(2^n - 1)/3 for all even n {n(mod2)=0}, this will generate an odd number that, when applying the 3k+1 rule, will generate an even number of the form 2^m.

What is upward convergence?

upward convergence is when at each step of the Collatz conjecture the k-index is reduced in each iteration of the Collatz process. To exemplify this, we will use the odd numbers of the form ck=4+12(n-1).

These would be the k-indices to analyze.

n	K=1+4(n-1)
1	1
2	5

n	K=1+4(n-1)
3	9
4	13
5	17
6	21
7	25
8	29
9	33
10	37
11	41
12	45
13	49
14	53
15	57
16	61
17	65
18	69

For practical purposes, as an example, we will analyze the case for the k-index 17.

k-index	3x+1	3x+2	3x+3
0	1	2	3
1	4	5	6
2	7	8	9
3	10	11	12

k-index	3x+1	3x+2	3x+3
4	13	14	15
5	16	17	18
6	19	20	21
7	22	23	24
8	25	26	27
9	28	29	30
10	31	32	33
11	34	35	36
12	37	38	39
13	40	41	42
14	43	44	45
15	46	47	48
16	49	50	51
17	52	53	54

The statement of the Collatz conjecture tells us the following: “If it is odd, multiply by 3 and add 1; if it is even, just divide by two”

for our case, 17 is odd, so we multiply by 3x+1 and get 52, 52 is even and we divide by two, following the rules defined previously j=floor(17/2)=8, so 3j+2 would be 26, 26 is even so k=floor(8/2)=4 and 3j+1 would be equal to 13, which is odd.

k-index	3x+1	3x+2	3x+3
1			
2			
3			

k-index	3x+1	3x+2	3x+3
4	13		
5			
6			
7			
8		26	
9			
10			
11			
12			
13			
14			
15			
16			
17	52		

Now 13 is odd, we multiply by 3 and add 1, this gives us 40, j=floor(13/2)=6 so 3j+2 is equal to 20, 20 is even, k=floor(6/2)=3, then 3k+1 is equal to 10, 10 is even so j=floor(3/2)=1, then 3j+2 is equal to 5

k-index	3x+1	3x+2	3x+3
0			
1			5
2			
3	10		
4			

k-index	3x+1	3x+2	3x+3
5			
6			20
7			
8			
9			
10			
11			
12			
13	40		
14			
15			
16			
17			

Now 5 is odd, so we multiply by 3 and add 1, this gives us 16 (if the index is odd the result of applying the Collatz rules will always give us an even number), j=floor(5/2)=2, then 3j+2 is equal to 8, 8 is even so k=floor(2/2)=1, then 3k+1 is equal to 4 four is even and we divide by 2, 2 is even and we divide by 1, 1 is odd and we multiply by 3 and add 1, we have reached the Collatz cycle.

k-index	3x+1	3x+2	3x+3
0			
1		5	
2			8
3			
4			

k-index	3x+1	3x+2	3x+3
5	16		
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			

This is “upward convergence,” no k-index was greater than the initial k-index of 16.

When the numbers take the form ck=10+12(n-1) or a number of the form ck=10+12(n-1) appears in some iteration, the upward convergence is slower, but it will eventually converge.

For example, we can start with a number of the form ck=4+12(n-1) that is divisible an odd number of times by 2.

as an example we take the k-index 25, by the rules described above we already know without calculating that 3x+1 is even and in our case it is 76, 76 is even, j=floor(25/2)=12, 3j+2 is equal to 38, 38 is even, k=floor(12/2)=6, 3k+1 is equal to 19, 19 is odd but it is of the form k=3+4(n-1) so it will generate a number of the form ck=10+12(n-1). 19 times 3 plus 1 is equal to 58, j=floor(19/2)=9, then 3j+2 is equal to 29, 29 is odd, but at the same time it is a k-index greater than the initial k-index of 25. 25 does not converge upward in the first iterations. 29 times 3 plus 1 is equal to 88, 88 is even, j=floor(29/2)=14, 3j+2=44, 44 is even, k=floor(14/2)=7, 3k+1=22, 22 is even, j=floor(7/2)=3, 3j+2 is equal to 11, 11 is odd so 3k+1 is even and is equal to 34, j=floor(11/2)=5, 3j+2 is equal to 17, 17 is odd and 3k+1 is even and equal to 52, which we know from the previous example that it will quickly converge to 4, 2, 1.

The Collatz conjecture is true.

The Collatz conjecture is true, since it is essentially a cyclical process that eventually alternates between numbers of the form 3k+1 and 3j+2, and this will always occur, since as soon as we find an odd number we will apply the 3x+1 rule and at that point we will always essentially oscillate between two columns of numbers.

On the other hand, the number ck=4+12(n-1), as the reduction process progresses, the k-index will decrease by 4 each time we find a prime number and are in an upward convergent cycle, so that in each step (n-1) will approach zero, until n=1.