A Novel Framework for Analyzing Prime Distribution and Weak Fermat's Conjecture.

Gilberto Augusto Carcamo Ortega • 10 Aug 2025

read paper. I couldn't include the images and certain features in the LaTeX and the PDF is a slightly more elaborate version to better understand the concepts.

The paper in English can be found here: Download paper (English).

El paper en español se puede encontrar aquí: Descargar paper (español).

The URL for the English paper is: https://drive.google.com/file/d/1vRtEOQpaylt4bJJL4RNDp1DXxoE0S68S/view?usp=drive_link.

author: Gilberto Augusto Carcamo Ortega.

Analysis of the Distribution of Prime Numbers on the Roulette Wheel

Results

Let's analyze the distribution of prime numbers within the real numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, ..., n-1, n, n+1.

Distribution of Prime Numbers

1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30

Distribution of Prime Numbers

31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	49	50
51	52	**53**	54	55	56	57	58	**59**	60
61	62	63	64	65	66	**67**	68	69	70
71	72	**73**	74	75	76	77	78	**79**	80
81	82	**83**	84	85	86	87	88	**89**	90
91	92	93	94	95	96	**97**	98	99	100

If we take a series of natural numbers, prime numbers appear in positions that coincide with the specific number being examined. For example, the first prime number appears in the 2nd position of the series of natural numbers, the second prime number in the 3rd position, and the n-th prime in the n-th position of the series of natural numbers. This is the simplest series to analyze (assuming it starts at $n = 1$):

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, \ldots, p, q.$$

If we analyze the differences between the terms, we do not find any visible pattern or a simple way to generate them. Therefore, at first, no periodicity is observed.
Now, let's distribute the first prime numbers into three columns, as in the canonical triplets. Mathematically, this is equivalent to three sets that do not contain each other, or three disjoint series:

Distribution of **canonical triplets**

columna 1	Columna 2	columna 3
3n+1	3n+2	3n+3
1	2	3
4	5	6
7	8	9
10	11	12

columna 1	Columna 2	columna 3
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
28	29	30
31	32	33
34	35	36

Beyond the first row, all rows have at most a single prime number. The pattern seems to be alternating, although it breaks in certain rows.

Rule number 1

Now we will define a rule that arises from the analysis of a simple strategy for playing roulette: betting on the number opposite to the last number played. If we follow this rule, we will realize that the opposite of an odd number is an even number one unit larger, and that every even number is opposite to an odd number one unit smaller.

"Every prime number in a row must always be accompanied by an even number to its right."

Rule number 2

"The third column only contains one prime number, and that prime number is 3, which occurs when n=0."

Now let's group the numbers in Odd-Even pairs

Distribution of **canonical triplets**

columna 1	Columna 2	columna 3
3n+1	3n+2	3n+3
1	2	3
4	5	6
7	8	9

columna 1	Columna 2	columna 3
10	11	12
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
28	29	30
31	32	33
34	35	36

Now, on this new arrangement, let's mark the prime numbers in red.

Distribution of **canonical triplets**

columna 1	Columna 2	columna 3
3n+1	3n+2	3n+3
1	**2**	**3**
4	**5**	6
7	8	9
10	**11**	12
13	14	15
16	**17**	18
19	20	21
22	**23**	24
25	26	27
28	**29**	30

columna 1	Columna 2	columna 3
31	32	33
34	35	36

From this new arrangement, the following conclusions can be drawn:

- **Each row can only contain a single prime number.**
- **The gaps of prime numbers are zones where the triplets are formed by composite numbers.**
- **The number of prime numbers in any set of numbers will be less than 1/3 of the total number of elements that make up the set.** Let
$$\mathrm{li}(x) = \int_2^x \frac{1}{\ln(t)} dt$$
be the logarithmic integral (the prime-counting function), if we evaluate the integral for example up to $n = 100$ we will realize that the percentage of prime numbers is never greater than 1/3.

Approximate values of $\mathrm{li}(x)$ and $\mathrm{li}(x)/x$

x	$\mathrm{li}(\mathbf{x})$ **(approximate)**	$\frac{\mathrm{li}(\mathbf{x})}{\mathbf{x}}$ **(approximate)**
10	5.1	0.51
20	7.2	0.36
30	9.4	0.31
40	11.5	0.29
50	13.8	0.28
60	16.1	0.27
70	18.5	0.26
80	20.8	0.26
90	23.2	0.26
100	25.6	0.26

When the numbers are grouped into three columns, a set of canonical progressions or single-variable equations emerge (there may be better definitions, but the simplest are these three):

- Column 1: $f(n) = 3n + 1$
- Column 2: $g(n) = 3n + 2$
- Column 3: $h(n) = 3n + 3$

Later, we will use these three equations as functions of x, y, and z.

Theorem of the Triplets.

"The ordered set of natural numbers can be considered an ordered set of points in the form $[f(x), g(y), h(z)]$, where x, y, and z take on real and integer values."

Analysis of Triplets.

When reorganizing numbers into triplets, it is evident that a gap is created when triplets of composite numbers appear. This, in itself, is not very helpful, but if we reflect on it, we can notice that between two triplets of composite numbers, or between groups of triplets of composite numbers, there must be at least one prime number. Thus, locating these triplets is of vital importance to determine where a prime number is or will be located, or failing that, where not to look.
The simplest triplet to analyze is the odd-even one, where the odd number ends in 5 and the even number ends in 6 (a multiple of two). However, due to the organization of the columns, finding where the numbers ending in 5 appear is sufficient.
Let's analyze the distribution of triplets to determine patterns:

In the first column, every number divided by $3x + 1$ has a remainder of 1; in the second, every number divided by $3x + 2$ has a remainder of 2; and in the third, every number divided by $3z + 3$ has a remainder of 0.
Each row alternates a quite distinct and obvious pattern (odd and even), and based on this pattern, we can analyze the distribution of triplets.
For a row to be a gap of prime numbers, its three elements must be composite numbers or, failing that, they could all be even numbers. However, according to the distribution of canonical triplets, there can only be two even numbers per row.
We must also consider that all elements in the third column are multiples of

three, so any number in that column will be composite.
Therefore, we only need to focus on analyzing columns 1 and 2. Now let's observe the following distribution of canonical triplets.

1 mod(3)	**2 mod(3)**	**0 mod(3)**
$f(x) = 3x + 1$	$g(y) = 3y + 2$	$h(z) = 3z + 3$
$1 \bmod (2)$	$0 \bmod (2)$	$1 \bmod (2)$
$0 \bmod (2)$	$1 \bmod (2)$	$0 \bmod (2)$
$1 \bmod (2)$	$0 \bmod (2)$	$1 \bmod (2)$
$0 \bmod (2)$	$1 \bmod (2)$	$0 \bmod (2)$
$1 \bmod (2)$	$0 \bmod (2)$	$1 \bmod (2)$
$0 \bmod (2)$	$1 \bmod (2)$	$0 \bmod (2)$
$1 \bmod (2)$	$0 \bmod (2)$	$1 \bmod (2)$
$0 \bmod (2)$	$1 \bmod (2)$	$0 \bmod (2)$
$1 \bmod (2)$	$0 \bmod (2)$	$1 \bmod (2)$
$0 \bmod (2)$	$1 \bmod (2)$	$0 \bmod (2)$
$1 \bmod (2)$	$0 \bmod (2)$	$1 \bmod (2)$
$0 \bmod (2)$	$1 \bmod (2)$	$0 \bmod (2)$
$1 \bmod (2)$	$0 \bmod (2)$	$1 \bmod (2)$
$0 \bmod (2)$	$1 \bmod (2)$	$0 \bmod (2)$
$1 \bmod (2)$	$0 \bmod (2)$	$1 \bmod (2)$
$0 \bmod (2)$	$1 \bmod (2)$	$0 \bmod (2)$

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Left Table			**Center Table**			**Index**
1-7 1	2	3	1	2	3	0
4	5	6	4	5	6	1
7	8	9	7	8	9	0
10	11	12	10	11	12	3
13	14	15	13	14	15	4
16	17	18	16	17	18	5
19	20	21	19	20	21	6
22	23	24	22	23	24	7
25	26	27	25	26	27	8
28	29	30	28	29	30	9
31	32	33	31	32	33	10
34	35	36	34	35	36	11
37	38	39	37	38	39	12
40	41	42	40	41	42	13
43	44	45	43	44	45	14
46	47	48	46	47	48	15
49	50	51	49	50	51	16
52	53	54	52	53	54	17
55	56	57	55	56	57	18

Left Table			Center Table			Index
58	59	60	58	59	60	19
61	62	63	61	62	63	20
64	65	66	64	65	66	21
67	68	69	67	68	69	22
70	71	72	70	71	72	23
73	74	75	73	74	75	24
76	77	78	76	77	78	25
79	80	81	79	80	81	26
82	83	84	82	83	84	27
85	86	87	85	86	87	28
88	89	90	88	89	90	29
91	92	93	91	92	93	30
94	95	96	94	95	96	31
97	98	99	97	98	99	32
100	101	102	100	101	102	33
103	104	105	103	104	105	34
106	107	108	106	107	108	35
109	110	111	109	110	111	36
112	113	114	112	113	114	37
115	116	117	115	116	117	38
118	119	120	118	119	120	39
121	122	123	121	122	123	40
124	125	126	124	125	126	41

Left Table			Center Table			Index
127	128	129	127	128	129	42
130	131	132	130	131	132	43
133	134	135	133	134	135	44
136	137	138	136	137	138	45
139	140	141	139	140	141	46
142	143	144	142	143	144	47

By analyzing the pattern where the pair of numbers ending in 5 and 6 appears, it's possible to demonstrate that the progression of numbers 8,11,18,21,28,31,41,... is given by two series.
For numbers of the form $3x+1$, the elements where $3x+1$ ends in 5 only occur when $n = 10K + 8$, where K is an integer.
For numbers of the form $3y+2$, a number ending in 5 will occur when $n = 10K + 1$.
Therefore, the progression of numbers 8,11,18,21,28,31,41,... is given by the following relationship: $(3x + 1 \mid n = 10K + 8)$, $(3x + 2 \mid n = 10K + 1)$
For the same value of K, two pairs of values are obtained.

K	$10k+8$	$10k+1$	$3x+1$	$3y+2$
0	8	1	25	5
1	18	11	55	34
2	28	21	85	64
3	38	31	115	94
4	48	41	145	124
5	58	51	175	154
6	68	61	205	184
7	78	71	235	214
8	88	81	265	244

K	$10k + 8$	$10k + 1$	$3x + 1$	$3y + 2$
9	98	91	295	274
10	108	101	325	304

Other triplets or gaps exist that present other patterns, such as:

- $3x + 1 = 49, 3y + 2 = 50, 3z + 3 = 51$, numbers ending in 9 and 0
- $3x + 1 = 76, 3y + 2 = 77, 3z + 3 = 78$, numbers ending in 7 and 8
- $3x + 1 = 91, 3y + 2 = 92, 3z + 3 = 93$, numbers ending in 1 and 2
- $3x + 1 = 133, 3y + 2 = 144, 3z + 3 = 145$, numbers ending in 3 and 4

Conjecture:

"A simple and straightforward series must exist that defines the indices where prime numbers are found. However, the series that indicates the distribution of prime numbers must be given by more than two parametric equations that define their indices."

Definition of the product of two real numbers: Product of two real numbers / Product of two prime numbers

Given the canonical equations:

- Column 1: $f(x) = 3x + 1$
- Column 2: $g(y) = 3y + 2$
- Column 3: $h(z) = 3z + 3$

We can conclude that the product of two integers is the result of multiplying two of these three canonical equations.
A number squared (a minimum condition, although there is a more complete condition that involves the multiplication of the prime factors of two natural numbers) is a number such that:

- $F(x) = f(x)^2$,
- $G(y) = g(y)^2$,

- $H(z) = h(z)^2$

If we take the product of two prime numbers p and q such that $p \neq q$ and both are different from 3, we obtain the following hyperbola (when the two numbers to be multiplied have the same canonical form, a parabola is obtained):

$$(3x+1)(3y+2) = K_p$$

where K_p is the product of p and q.
More generally: "The product of all prime numbers p and q defines all the level curves of the function:" $9xy + 6x + 3y + 2 = K_P$

- Every equation of the form $9xy + 6x + 3y + 2 = K_p$ has a unique positive integer solution.
- All points on the curve $9xy + 6x + 3y + 2 = K_p$ are constant and equal to the product of p and q.

Weak Proof of Fermat’s Theorem.

Conic sections are curves that are studied in basic geometry courses at the beginning of high school. It is fascinating how a circle could define one of the most important mathematical problems in the world, when **Fermat** wondered if Pythagorean triplets could be obtained for exponents greater than 2.
I consider this a demonstration, perhaps not a formal one, but it could have been the path Fermat took to outline his famous theorem, which was impossible to write in the margin of a sheet.
I believe that Fermat had to intuit, or even find a way, that for the equation $x^n + y^n = z^n$ to have a solution in higher degrees, it had to belong to a family of equations resulting from the intersection of a surface by a plane.
Given the inability to find such closely related families of curves in higher degrees, the only logical conclusion is the impossibility of solving the problem for degrees greater than two. This is a **very difficult conjecture to prove**.
In my case, the distribution of my arithmetic progressions and the products of their terms as functions only generate **parabolas** and **hyperbolas**.
Another set of relations and distributions would be required to generate **circles** and **ellipses** in order to address more complete solutions, since my canonical triplet distribution currently only generates two types of conics.

Arithmetic Progressions and Prime Numbers

Consider the simple arithmetic progression $(n+1)$, where n takes non-negative integer values ($n = 0, 1, 2, ...$). This progression generates all natural numbers.
If we define $c = n+1$, then $c^2 = (n+1)^2$ is a second-degree polynomial in n.
To analyze the distribution of prime numbers, we define three disjoint arithmetic progressions:

- $a(n) = 3n+1$
- $b(n) = 3n+2$
- $c(n) = 3n+3$

More generally, we can use independent variables:

- $f(x) = 3x+1$
- $g(y) = 3y+2$
- $h(z) = 3z+3$

Consider the product of two terms of these progressions, for example, $K_p = f(x)g(y)$. This product generates a quadratic curve. Specifically, if we choose terms from two different progressions (for example, $f(x)$ and $g(y)$), K_p represents a hyperbola. If we choose two terms from the same progression, we get a parabola.
Example: $K_p = (3x+1)(3y+2)$. We choose this product because the only prime number in $(3n+3)$ occurs when n=0.

Conditions for Square Numbers

For $K_p = c^2$, where c is a natural number, the following conditions must be met:

- K_p must be a perfect square ($K_p = p^2$).
- K_p must be a perfect square ($K_p = q^2$).

If $K_p = p \cdot q$, where p and q are natural numbers, then the prime factors of p and q must have even exponents in their prime decomposition. That is:

- $p = 2^{n_1} \cdot 3^{n_2} \cdot 5^{n_3} \cdot \dots = \prod_{i=1}^{k} P_i^{n_i}$
- $q = 2^{m_1} \cdot 3^{m_2} \cdot 5^{m_3} \cdot \dots = \prod_{j=1}^{k} Q_j^{n_j}$

Where $n_i + m_j$ is an even number for every index i and j.
If these conditions are met, then $(3x+1)(3y+2) = c^2$. More generally, the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = c^2$ has positive integer solutions. In the quadratic case, all conics are classified under projective transformations.
Conic sections are generated by the intersection of a plane with a cone (Figure **??**). If the plane is parallel to the axis of revolution (the y-axis), the conic section is a hyperbola. If the plane is parallel to the generatrix, the conic section is a parabola. If the plane is perpendicular to the axis of revolution, the conic section is a circle. If the plane intersects one sheet at an angle with the axis (other than 90°), then the conic section is an ellipse.

Generalization to Higher Exponents

To obtain natural numbers of the form $x^n + y^n = c^n$, we use the trivial arithmetic progression $(n+1)$. So, $c^n = (n+1)^n$, which is a polynomial of degree n:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

For degrees higher than 2, the intersection curves do not belong to a single family like the conics. They can have different genera, singularities, and irreducible components. Therefore, there is no general way to reduce $f(x)$ to $x^n + y^n = c^n$, which suggests that there are no positive integer solutions for $n > 2$ since $f(x)$ has positive integer solutions and from $f(x)$ I cannot reduce to $x^n + y^n = c^n$.
The results obtained in section 1.5 suggest that every number that is the product of two prime numbers can be represented by two equations, hyperbolas and parabolas, both conic sections, which makes it probable that all curves of the family $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = c^2$ have real and integer solutions.
Furthermore, the distribution of canonical triplets establishes that $9xy + 6x + 3y + 2 = K_p$ will always have real solutions as long as K_p is the product of two integers, and as we have seen in section 2.1, every number has a prime power decomposition.

Weak Proof. Proof based on the distribution of canonical triplets.

the equation $x^n + y^n = z^n$ only makes sense for the family of conic curves for $n = 2$ since the relations given in the canonical triplet distribution only allow for prime numbers of the form $(3x+1)$ and $(3y+2)$ which generates conic equations. The canonical form $(3z+3)$ or $3(z+1)$ only has one prime number

and it occurs when $z = 0$.
Multiplying by the canonical form $3(z+1)$ is the equivalent of multiplying by any number given by $(3x+1)$ and $(3y+2)$, since $(z+1)$ contains all natural numbers and therefore contains $(3x+1)$ and $(3y+2)$, so only the possible combinations of these two products $(3x+1)$ and $(3y+2)$ can generate an integer product of prime numbers.

Distribution of **canonical triplets**. Canonical form.

columna 1	Columna 2	columna 3
3x+1	3y+2	3z+3
1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
28	29	30
31	32	33
34	35	36

Fermat's Weak Conjecture.

The equation $x^n + y^n = z^n$ only has a solution for $n = 2$ given the impossibility of forming a product of three terms without one of the three terms being a multiple of the other two. The equation $x^n + y^n = z^n$ only makes sense if the curves it generates are conic sections. The same happens in higher degrees since we would have to multiply by elements of the same canonical form n times.

An Approximation for Finding Prime Numbers Based on the Distribution of Canonical Triplets

For the development of this method, we will use the distribution of canonical triplets and the equation $9xy + 6x + 3y + 2 = K_p$.

What happens at $x = 0$ and $y = 0$? Approximation by Solving a Quadratic

By considering the value of $x = 0$ in the equation $9xy + 6x + 3y + 2 = K_p$, we get the following result:

$$y = \frac{K_p - 2}{3} \tag{1}$$

Similarly, by evaluating the equation $9xy + 6x + 3y + 2 = K_p$ at the point $y = 0$, we get the following result:

$$K_p = 6x + 2 \tag{2}$$

If we substitute equation 8 into equation 7, we get the following approximation:

$$y = 2x \tag{3}$$

Let's take the equation $9xy + 6x + 3y + 2 = 65$ as an example, where K_p is the product of the prime numbers $q = (3y + 1) = 5$ and $p = (3x + 2) = 13$. For this particular case, we can find the values of x and y by solving the following equation which is obtained by using 9 in $9xy + 6x + 3y + 2 = K_p$:

$$18x^2 + 12x + 2 = K_p \tag{4}$$

To solve the quadratic equation $18x^2 + 12x + 2 = K_p$, we first rewrite it in the standard form $ax^2 + bx + c = 0$:

$$18x^2 + 12x + (2 - K_p) = 0$$

We identify the coefficients: $a = 18$, $b = 12$, and $c = 2 - K_p$. We apply the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

$$x = \frac{-(12) \pm \sqrt{(12)^2 - 4(18)(2 - K_p)}}{2(18)}$$

$$x = \frac{-12 \pm \sqrt{144 - 72(2 - K_p)}}{36}$$

$$x = \frac{-12 \pm \sqrt{144 - 144 + 72K_p}}{36}$$

$$x = \frac{-12 \pm \sqrt{72K_p}}{36}$$

We can simplify the square root $\sqrt{72K_p}$: $\sqrt{72K_p} = \sqrt{36 \cdot 2 \cdot K_p} = 6\sqrt{2K_p}$ Substituting this back into the equation for x:

$$x = \frac{-12 \pm 6\sqrt{2K_p}}{36}$$

Finally, we divide each term by 6 to simplify:

$$x = \frac{-2 \pm \sqrt{2K_p}}{6}$$

Thus, the solutions for x are:

$$x_1 = \frac{-2 + \sqrt{2K_p}}{6}$$

$$x_2 = \frac{-2 - \sqrt{2K_p}}{6}$$

These are the solutions for x in terms of K_p. For the solutions to be real, the term under the square root must be non-negative, i.e., $2K_p \geq 0$, which implies $K_p \geq 0$.

Due to the nature of the problem we are analyzing, we will only consider the values of x given by

$$x = \frac{-2 + \sqrt{2K_p}}{6} \tag{5}$$

And by resorting to equation 9, we conclude the value of y:

$$y = \frac{-2 + \sqrt{2K_p}}{3} \tag{6}$$

We substitute the value of $K_p = 65$ into both equations. First, we calculate $\sqrt{2 \cdot 65} = \sqrt{130} \approx 11.40175$:

$$\sqrt{130} \approx 11.40175$$

$$x = \frac{-2 + \sqrt{130}}{6} \approx \frac{-2 + 11.40175}{6} = \frac{9.40175}{6} \approx 1.56696$$

Therefore:

$$\mathbf{x \approx 1.56696}$$

$$y = \frac{-2 + \sqrt{130}}{3} \approx \frac{-2 + 11.40175}{3} = \frac{9.40175}{3} \approx 3.13392$$

Therefore:

$$\mathbf{y \approx 3.13392}$$

The approximate values for x and y for $K_p = 65$ are $x \approx 1.56696$ and $y \approx 3.13392$.
Knowing that $K_p = 65 = p \cdot q = 5 \cdot 13$, we will now proceed to solve the equations

$$(3x + 1) = 13$$

$$(3y + 2) = 5$$

For x:

$$x = \frac{12}{3} = 4$$

For y:

$$y = \frac{3}{3} = 1$$

Distribution of **canonical triplets**. Canonical form.

Column 1	Column 2	Column 3
3x+1	3y+2	3z+3
1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
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Fermat's Weak Conjecture.

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By considering the value of $x = 0$ in the equation $9xy + 6x + 3y + 2 = K_p$ we get the following result:

$$y = \frac{K_p - 2}{3} \tag{7}$$

Similarly, by evaluating the equation $9xy + 6x + 3y + 2 = K_p$ at the point $y = 0$ we get the following result:

$$K_p = 6x + 2 \tag{8}$$

If we replace equation 8, in equation 7 we obtain the following approximation:

$$y = 2x \tag{9}$$

Approximation at Infinity.

Let's consider the canonical forms:

- $f(n) = 3 \cdot n + 1$
- $g(n) = 3 \cdot n + 2$
- $h(n) = 3 \cdot n + 3$

If we take limits in the following way:

- $\lim_{n \to \infty} f(n) = lim_{n \to \infty} 3 \cdot n + 1 = 3n$
- $\lim_{n \to \infty} g(n) = lim_{n \to \infty} 3 \cdot n + 2 = 3n$
- $\lim_{n \to \infty} h(n) = lim_{n \to \infty} 3 \cdot n + 3 = 3n$

From this result at infinity we might be tempted to use the following equation $(3n)^2 \approxeq k_p$, solving this equation we obtain the following approximation:

$$n \approxeq \frac{\sqrt{K_p}}{3} \tag{10}$$

This choice is logical since the factorization of prime numbers is fundamental in cryptography, especially in algorithms like RSA. The security of these algorithms is based on the difficulty of factoring large numbers that are the product of two prime numbers.
This choice is made under the assumption that our prime numbers are extremely large. Now that we have this equation we can obtain a better search range by using equation 9 together with equation 10 in the following way:

We will assume that in equation 10 n is equal to x $(n = x)$. From that approximation we obtain the point P_{1a}

$$P_{1a} = \left(\frac{\sqrt{K_p}}{3}, \frac{2 \cdot \sqrt{K_p}}{3} \right) \tag{11}$$

To find the next point we will say that in equation 10 n is equal to y $(n = y)$. From that approximation we obtain the point P_{1b}

$$P_{1b} = \left(\frac{\sqrt{K_p}}{6}, \frac{\sqrt{K_p}}{3} \right) \tag{12}$$

Given that both points must be considered as search ranges we would be left with a range that includes the smallest and the largest number, so for practical purposes we define the following range.

$$R = \left(\frac{\sqrt{K_p}}{6}, \frac{2 \cdot \sqrt{K_p}}{3}\right) \tag{13}$$

K-indices and the sums to 1 and sums to 2 of the $3x + 3$ column

Days ago I was analyzing the ways in which a number can be prime. We already know that a prime number can take the following forms $\boldsymbol{6m+1}$ and $\boldsymbol{6m-1}$. That also implies that these numbers are by definition **odd**.
From that idea my question arose (which is obvious): how should a number end to be odd? The answer is simple: every odd number must end in **1, 3, 5, 7, or 9**.

For this I define the sum to 1 of the k-indices as follows

The reduction to a single digit is the process of repeatedly summing the digits of a number until a single digit (from 1 to 10) is obtained.

Sum to 2 of 3x+1 and 3x+3

The reduction to two digits involves repeatedly summing the digits of a number until the result is a two-digit number (between 10 and 30, inclusive), or a single digit if the sum never reaches two digits.
As an additional gift every number of the form $3x + 1$ where $x = 10k + 8$ and $3x + 2$ such that $x = 10k + 1$ will always end in 5 and will be composite by definition, except for the number 5, which generates two barriers.
Then I wondered what that would look like in my array of triplets $\{3x + 1, 3x + 2, 3x + 3\}$.

K indices

K index	$3x+1$	$3x+2$	$3x+1$	$3x+2$	$3x+1$	$3x+2$	$3x+1$	$3x+2$	$3x+1$
	1	5	7	1	3	7	9	3	5
	$10k$	$10k+1$	$10k+2$	$10k+3$	$10k+4$	$10k+5$	$10k+6$	$10k+7$	$10k+8$

K index	$3x+1$	$3x+2$	$3x+1$	$3x+2$	$3x+1$	$3x+2$	$3x+1$	$3x+2$	$3x+1$
0	0	1	2	3	4	5	6	7	8
1	10	11	12	13	14	15	16	17	18
2	20	21	22	23	24	25	26	27	28
3	30	31	32	33	34	35	36	37	38
4	40	41	42	43	44	45	46	47	48
5	50	51	52	53	54	55	56	57	58
6	60	61	62	63	64	65	66	67	68
7	70	71	72	73	74	75	76	77	78
8	80	81	82	83	84	85	86	87	88
9	90	91	92	93	94	95	96	97	98
10	100	101	102	103	104	105	106	107	108
11	110	111	112	113	114	115	116	117	118
12	120	121	122	123	124	125	126	127	128
13	130	131	132	133	134	135	136	137	138
14	140	141	142	143	144	145	146	147	148
15	150	151	152	153	154	155	156	157	158
16	160	161	162	163	164	165	166	167	168
17	170	171	172	173	174	175	176	177	178
18	180	181	182	183	184	185	186	187	188
19	190	191	192	193	194	195	196	197	198
20	200	201	202	203	204	205	206	207	208

k	Terminaciones									
2-11	1	5	7	1	3	7	9	3	5	9
	$3x+1$	$3x+2$	$3x+1$	$3x+2$	$3x+1$	$3x+2$	$3x+1$	$3x+2$	$3x+1$	$3x+2$
0	1	5	7	11	13	17	19	23	25	29
1	31	35	37	41	43	47	49	53	55	59
2	61	65	67	71	73	77	79	83	85	89
3	91	95	97	101	103	107	109	113	115	119
4	121	125	127	131	133	137	139	143	145	149
5	151	155	157	161	163	167	169	173	175	179
6	181	185	187	191	193	197	199	203	205	209
7	211	215	217	221	223	227	229	233	235	239
8	241	245	247	251	253	257	259	263	265	269
9	271	275	277	281	283	287	289	293	295	299
10	301	305	307	311	313	317	319	323	325	329
11	331	335	337	341	343	347	349	353	355	359
12	361	365	367	371	373	377	379	383	385	389
13	391	395	397	401	403	407	409	413	415	419
14	421	425	427	431	433	437	439	443	445	449
15	451	455	457	461	463	467	469	473	475	479
16	481	485	487	491	493	497	499	503	505	509
17	511	515	517	521	523	527	529	533	535	539
18	541	545	547	551	553	557	559	563	565	569
19	571	575	577	581	583	587	589	593	595	599
20	601	605	607	611	613	617	619	623	625	629

Every number that ends in 1 and is of the form 3x+1 such that x=10k, will be composite when its sum to 1 of its k-index is equal to 3, 9, 10, its sum to two of 3x+1 is equal to 4, 28 and its sum to two of 3x+3 is equal to 6, 30

index	10k	3x+1	3x+3	sum index	sum 3x+1	sum 3x+3	prime
37	370	1111	1113	10	4	6	C
333	3330	9991	9993	9	28	30	C
334	3340	10021	10023	10	4	6	C
370	3700	11101	11103	10	4	6	C
633	6330	18991	18993	3	28	30	C
666	6660	19981	19983	9	28	30	C
963	9630	28891	28893	9	28	30	C
966	9660	28981	28983	3	28	30	C
993	9930	29791	29793	3	28	30	C
999	9990	29971	29973	9	28	30	C

Legendre’s Conjecture: Considerations for the Proof

For the proof of Legendre’s conjecture, we will consider the following:

- The set of natural numbers $\mathbb{N}$ is infinite.
- The subset of prime numbers is also infinite.
- Within the **Distribution of Canonical Triplets** there is only one triplet of numbers that contains two primes: $\{1, 2, 3\}$.

Form $3n + 1$	**Form** $3n + 2$	**Form** $3n + 3$
1	2	3

Form $3n+1$	**Form** $3n+2$	**Form** $3n+3$
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
28	29	30
31	32	33
34	35	36

- Every triplet with an index $n \geq 1$ can contain at most one prime number.
- It is possible to find triplets composed of three composite numbers. The parity of the triplets follows a regular pattern, where the forms $3x+1$ and $3x+2$ alternate their parity, and the form $3x+3$ is always even.

Remainder 1 mod(3)	**Remainder 2 mod(3)**	**Remainder 0 mod(3)**
Form $3x+1$	**Form** $3y+2$	**Form** $3z+3$
$1 \pmod 2$	$0 \pmod 2$	$1 \pmod 2$
$0 \pmod 2$	$1 \pmod 2$	$0 \pmod 2$

- Understanding that the set of natural numbers is infinite, it is possible to find a number K_N which is the product of two natural numbers such that $K_N = p \cdot q$. In this context, p and q may or may not be prime numbers. These numbers p and q can be expressed in the following forms: $p = 3k+1$ and $q = 3k+2$, which coincides with the canonical triplets.
- The canonical curve, which is the product of the conic forms $(3x+1)$ and $(3y+2)$, is $k_N = (3x+1)(3y+2)$.

- Between two triplets of composite numbers, there will always exist at least one prime number.

By observing the **Distribution of Canonical Triplets** (Table 1), we can clearly see that for each row or triplet, there is only one prime number from index $k \geq 1$. Since prime numbers are infinite, there will always be a triplet with at least one prime at index k.

Statement of Legendre's Conjecture

Legendre's conjecture states that for every positive integer $n \in \mathbb{Z}^+$, there exists a prime number p such that:

$$n^2 < p < (n+1)^2$$

Proof: A Specific Case

We define the functions $f(x)$ and $g(x)$ and their squares $F(x)$ and $G(x)$:

$$f(x) = 3x + 1$$

$$g(x) = f(x) + 1 = 3x + 2$$

$$F(x) = (f(x))^2 = (3x+1)^2$$

$$G(x) = (g(x))^2 = (3x+2)^2$$

This definition is, in essence, the statement of Legendre's conjecture. The equations for $F(x)$ and $G(x)$ represent two parabolas.

As a first example, let's analyze the case where $x = 0$.
$F(0) = (3(0)+1)^2 = 1^2 = 1$ $G(0) = (3(0)+2)^2 = 2^2 = 4$ In this case, the definition of Legendre's conjecture is satisfied, since the prime numbers 2 and 3 are found in the interval $(1, 4)$.

Generalization

We define two sets, A and B:

1. Set A is composed of all values of the form $3x + 1$, where $x \in \mathbb{Z}$.
2. $$A = \{3x + 1 \mid x \in \mathbb{Z}\}$$
   Set B is composed of all values of the form $3y + 2$, where $y \in \mathbb{Z}$.
   $$B = \{3y + 2 \mid y \in \mathbb{Z}\}$$

Both sets, A and B, are **infinite** because x and y can take an infinite number of values in $\mathbb{Z}$.
Now, we define a new set, M, which contains the result of the multiplication of each element of A with each element of B:

$$M = \{(3x+1)(3y+2) \mid x, y \in \mathbb{Z}\}$$

Since A and B are infinite sets, set M is also infinite.

As M is infinite and contains all possible values of $K_N = (3x+1)(3y+2)$, there are an infinite number of equations of this form. These equations can cross the intervals of Legendre's conjecture, such as $[n^2, (n+1)^2]$. It can be concluded that for infinite combinations of products of numbers of the forms $(3x+1)$ and $(3y+2)$, there will always exist a point $[x, y]$ such that the values of y will be within the range $[n^2, (n+1)^2]$, thus verifying the conjecture for this particular case.

Legendre Function

The only function that satisfies Legendre's Conjecture under these conditions is:

$$H(x) = (3x+1-h)^2 \quad \text{such that} \quad -2/3 < h < -1/3$$

And the following inequality holds:

$$(3x+1)^2 < (3x+1-h)^2 < (3x+2)^2$$

such that:

$$(3x+1-h) = \sqrt{p}$$

This implies that:

$$(3x+1-h)^2 = \begin{cases} 6m+1 \\ 6m+5 \end{cases}$$

the only solution that satisfies this equation is when h is an irrational number. All the solutions of Legendre's hypothesis must be found in the interval.

$$-\frac{2}{3} < h < -\frac{1}{3}$$

.

Where $6m+1$ and $6m+5$ are odd numbers that satisfy the minimum conditions for a number to be prime.