Getz’s Automorphic forms Chapter 2

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See here for an intro on the transition from modular forms to automorphic forms. They are not really scary!!!

Relationship between automorphic representation and number theory

See Example 2.1 for an example of strong approximation: omitting one place change the image of $G(\mathbb{Q})$ to dense from discrete.

Section 2.2 is about how to equip $X(R)$ with a natural topology if R is a topological ring satisfying some axioms (this approach is first discovered by Conrad). Then we can give an alternative description of $G(\mathbb{A})$ using restricted direct product.

A hyperspecial subgroup is of the form $\mathcal{G}(\mathcal{O}_F)$ for a model $\mathcal{G}$ of G with reductive special fiber. Such subgroup exists if G is unramified over F.

Every maximal compact subgroup of $G(F)$ is of the form $\mathcal{G}(\mathcal{O}_F)$ for some smooth model $\mathcal{G}$. The adjoint group $G_{ad} := G/Z_G$ acts on G by conjugation, and the orbit of $G_{ad}(F)$ on $G(F)$ is in general larger than the orbit of $G(F)$. The set of hyperspecial subgroups is permuted transitively by the action of $G_{ad}(F)$ when G is split, but not $G(F)$ in general. Given a reductive group G over a global field F one can readily construct hyperspecial subgroups of $G(F_v)$ for all but finitely many v.

The next few propositions is about existence of models and how it refines description of $G(\mathbb{A})$ in terms of restricted direct product w.r.t. $\mathcal{G}(\mathcal{O}_{F_v})$. Note that instead of using this collection of maximal compact subgroups we can choose some ‘exotic’ collection of models chosen place by place, but then it won’t be functorial and in particular won’t commute with the diagonal embedding of $G(F)$ into $G(\mathbb{A})$ and $\Pi'_v G(F_v)$. For nonaffine scheme X we will use the restricted direct product bijection to define the topology on $X(\mathbb{A})$.

Section 2.5 is about when weak and strong approximation holds. Section 6 introduces adelic quotient. Section 2.7 talks about reduction theory (finding approximate fundamental domain for $G(F) \setminus G(\mathbb{A}_F)$.