Why should we believe Mordell’s conjecture aka Faltings’s theorem

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If the curve C is given by complete intersection of degree $(d_1, ...d_k)$ inside $\mathbb{P}^n$, then the genus is ≥ 2 iff $\sum d_i > n+1$ (see here for the formula for genus of complete intersection, which follows from the adjunction formula and the Euler exact sequence). Now a naive probabilistic argument shows that in this case $f_1(X) = ... = f_k(X) = 0$ is likely to have finitely many primitive solutions $X \in \mathbb{Z}^{n+1}$. This argument gives the generally believed heuristic for higher dimensional varieties: varieties with ample canonical bundle tend to have few points.