Quadratic twist of elliptic curve

J'ignore • 4 Sep 2025

Given an elliptic curve of the form $y^2 = x^3 + ax + b$ over F (so $a, b \in F$). For any $d \in F$, we can define an elliptic curve over $E^d := dy^2 = x^3 + ax + b$ over F. Note that E^d and E are isomorphic over $K := F(\sqrt{d})$ (isomorphism given by sending (x, y) on E to $(x, y/\sqrt{d})$). Note $\mathrm{rank} E(K) = \mathrm{rank} E(F) + \mathrm{rank} E^d(F)$. This is because of the exact sequence

$$0 \to E^d(F) \to E(K) \to E(F) \to V \to 0$$

where the middle map $E(K) \to E(F)$ is the trace map and $E(F)/2E(F)$ surjects onto V.