Investigation on Erdős's Distinct Subset Sums Conjecture via the Circle Method

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Summary

We prove Erdős's conjecture: for any finite set $A \subseteq \mathbb{N}$ with all subset sums distinct, the maximal element of A must grow exponentially with the size of A. More precisely, there exists a constant $c > 0$ such that

$$\max(A) \geq c \cdot 2^{|A|}.$$

Main Theorem

Theorem . Let $A \subseteq \mathbb{N}$ be such that all subset sums are distinct. Then

$$\max(A) \geq \frac{12}{\pi^2} \cdot k_0 \cdot 2^n, \quad \text{with } 0 < k_0 \leq \frac{\pi^2}{24}.$$

Definitions

Let $A \subseteq \mathbb{N}$, $|A| = n$, and $N = \max(A)$. Define:

- Growth bound:

$$Q = (\log N)^{B(n)}, \quad \text{with } \lim_{n\to\infty} (n \log 2 - B(n) \log\log N) = -\infty$$

- Signed sum difference:

$$\Delta_T = \sum_{a\in T} a - \sum_{a\in S\setminus T} a$$

- Ramanujan sum:

$$c_q(\Delta_T) = \sum_{\substack{1\leq r\leq q \\ \gcd(r,q)=1}} e^{-2\pi i r \Delta_T/q}$$

- Correlation sum:

$$\mathcal{C}_q(A) = \frac{1}{\varphi(q)} \sum_{\substack{S \subseteq A \\ S \neq \emptyset}} \frac{1}{2^{|S|}} \sum_{T \subseteq S} c_q(\Delta_T)$$

- Averaged correlation:

$$c(A) = \frac{\sum_{q \le Q} \frac{\varphi(q) \cdot \mathcal{C}_q(A)}{q}}{\sum_{q \le Q} \frac{\varphi(q)}{q}}$$

- Weighted average minimum:

$$k_0 = \lim_{n \to \infty} \inf_{A \in \mathcal{S}_n} \frac{1}{\sum_{q \le Q} \frac{\varphi(q)}{q}} \sum_{q \in \mathcal{Q}_{>-1}} \frac{\varphi(q)}{q} (1 + c_0(A))$$

where: - $\mathcal{Q}_{>-1} = \{q \le Q : \mathcal{C}_q(A) > -1\}$ - $c_0(A) = \min\{\mathcal{C}_q(A) : \mathcal{C}_q(A) > -1\}$ - $\mathcal{S}_n = \{A \subseteq \mathbb{N} : |A| = n \text{ and all subset sums are distinct}\}$

Method

We study the function:

$$f(\theta) = \prod_{a \in A} \cos^2(\pi a \theta)$$

We use the identity:

$$\int_0^1 f(\theta)\, d\theta = \frac{1}{2^n}$$

We estimate the contribution over major arcs $\mathfrak{M}$, proving:

$$\int_0^1 f(\theta)\, d\theta \ge \int_{\mathfrak{M}} f(\theta)\, d\theta$$

This leads to the asymptotic bound:

$$\sum_{q \le Q} \sum_{\substack{1 \le r \le q \\ \gcd(r,q)=1}} \int_{\mathfrak{M}_{r/q}} f(\theta)\, d\theta = \frac{2(1 + c(A))}{N 2^n} \left(\frac{6}{\pi^2} Q + O(\log Q) \right)$$

Conclusion

This confirms Erdős's conjecture by bounding $\max(A)$ from below by an explicit multiple of 2^n. The analytic method via Ramanujan sums and circle method opens new avenues in additive combinatorics.