# Convolutional Discrete Calculus – Video Outline

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original link: https://functor.network/user/288/entry/168

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## PART 1: sequences, patterns, and discrete calculus

1. Problem: can you find the formula for a sequence?
    1. Try discrete differences
2. Problem: that's the pattern, but what's the formula?
    1. Write it in terms of antidifferences
    2. Antidifferences are indefinite sums up to a constant
    3. Write it in terms of sums
3. Problem: that's the sum formula, but what's the polynomial formula?
    1. Summing sequences sums their differences
    2. Multiplying a sequence by a constant multiplies its difference by a constant
    3. Use the geometry of rectangles to solve for the geometry of a triangle
4. Problem: that was hard, is there an easier way to find a more general polynomial formula?
    1. List off the difference for each monomial
    2. Rearrange the chart in terms of sums
    3. Solve for each sum
    4. Use the chart as a reference
5. Problem: can you find the level of an (easy) sequence?
    1. (introduce without level, then introduce level)
    2. Take differences until it's constant
6. Problem: can you find the level of a (medium) sequence?
    1. Differences don't work
    2. Sums and differences cancel out
    3. Sums are like negative differences
    4. Take sums till it's constant, and make that number negative
7. Problem: finding the level of a hard sequence
    1. Differences don't work, neither do sums
    2. The answer is 2.5, but getting there is complicated, save it for part 2

## PART 2: recurrence transformations, weight analysis, and functional roots

1. Problem: what exactly is a sequence's level?
    1. Put simply, the level of the sequence the degree of its polynomial expression
    2. Some functions, which aren't polynomials, have no level

2. Problem: what about sequences which have a degree but aren't polynomials?
    1. The polynomial definition works for polynomials, but clearly that's only part of a larger phenomenon
    2. The degree is how many sums/differences you need to apply to a constant to get a function
3. Problem: ok, but what exactly are sums and differences?
    1. Mess with variations on the difference definition to get a more general "recurrence transformation" phenomonon
    2. Reframe the sum as a recurrence transformation
    3. Use weight analysis to define higher order sums/differences as recurrence transformations
4. Problem: what does this tell us about level?
    1. $R-$Transforming two sequences adds their level
    2. You can find level by splitting into $R-$factors
    3. Level is just how many unit $R-$factors the sequence can be split into
5. Problem: why is the sum the unit?
    1. Easy answer: it lines up nicely with polynomials, but that's not very satisfying
    2. Try starting with a different base unit and base inverse
    3. Try finding the level of some sequences
    4. Everything still works, just scaled differently
    5. Interestingly, some sequences can now have non-integer level
    6. It actually is kind of arbitrary
6. Problem: could we have found the sum sequence from the double sum unit sequence?
    1. Easy answer: use the difference, but that's kind of cheating
    2. Use a table to visualize $R-$transforms
    3. Reverse the problem to solve for each item on the sides of the table
    4. Vocab: functional root
7. Problem: can we find the functional root of the sum sequence?
    1. Use the table method from before
    2. There's no easy pattern, but it works!
8. Problem: now can we return to the level of the hard sequence?
    1. Use our rules and some algebra to work out that the new sequence has level $\frac{1}{2}$
    2. Use our rules to figure out that the sequence from before has level 2.5
9. Problem: start with a level and find a sequence 3.125?
    1. Use the functional root technique
    2. Show that it works for any dyadic rational level
10. Problem: can you find sequences with more numbers $(\frac{1}{3}, \pi, i)$?
    1. Functional square roots won't help us, since $\frac{1}{3}$ is non-dyadic
    2. Functional cube roots could work for $\frac{1}{3}$, but calculating them is a hot mess
    3. Functional roots are basically hopeless for $\pi$
    4. No clue where to even start with $i$

5. This *is* possible, but getting there is complicated, save it for part 3

## PART 3: convolutions, derivatives and power series

1. Problem: can we find a pattern for the weights in the $\frac{1}{2}$ level sequence?
    1. Trying difference techniques won't work
    2. Try on just the numerator and denominator, that still doesn't work
    3. Try and reframe the problem as shifted and scaled copies
    4. Notice location addition and value multiplication
    5. Tangent to polynomials
    6. Reframe polynomial multiplication as shifted and scaled copies
    7. Notice location addition and value multiplication
    8. Show equivalence between functional root problem and the polynomial square root problem
    9. Demonstrate that the equivalence works by testing it on the second order sum sequence
2. Problem: ok, but how do you find a pattern?
    1. Use algebra to show

    $$ips(sum) = \frac{1}{(ips(dif))}$$

    2. Reframe the problem as power representation of

    $$\sqrt{\frac{1}{1-x}}$$

    3. Show that our manual calculations for the $\frac{1}{2}$ sequence work to approximate it
3. Problem: how does that help us? where's the pattern?
    1. Define an analytic function, and show that $\sqrt{\frac{1}{1-x}}$ is one
    2. Take: calculus says that every analytic function has a power series
    3. Take: the coefficients will be value, rate, rate of rate, etc
    4. Take: you can calculate rate by taking a derivative
    5. Take: you can find the derivative of any analytic function using these rules
    6. Demonstrate that evaluating derivatives of $\frac{1}{\sqrt{1-x}}$ produces values of the $\frac{1}{2}$ sequence
4. Problem: ok, but how do we get from there to an actual, closed form pattern?
    1. Unpack the derivative rules to show patterns in the functions
    2. Simplify the patterns for the special $x = 0$ case
    3. Derive a real pattern in closed form
    4. Review: to find a level $\frac{1}{2}$ sequence, we took the power series of $\frac{1}{(1-x)^{(1/2)}}$ and derived a pattern from the differentiation rules
5. Problem: can we return to find sequences with weird weights?

    1. For $\frac{1}{3}$, just take the series coefficients for $\frac{1}{(1-x)^{(1/3)}}$
    2. Test that it genuinely does work out to the functional cube root of $ips(sum)$
    3. For $\pi$, just take the series coefficients for $(1-x)^{-\pi}$
    4. You can't test it the same way, but it does work
6. Problem: what about the level i sequence?
    1. Complex exponentiation works with new rules
    2. We don't need them though, we can just plug and chug
    3. Deriving a pattern
    4. Finding the sequence
    5. Demonstrating that it works

## Outro

1. Credits:
    1. Mythologer's video
    2. Supware's video
    3. Morphocular's video
2. Unanswered questions:
    1. What actually is complex exponentiation?
    2. What other sequences have no level?
    3. What if we start with a level-less unit $R-$transformation?
3. For next time:
    1. Can we leverage the connection between differences and derivatives to extend our techniques to the continuous relm?