One reason that makes the axiomatic contruction of reals interesting.

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Why is the axiomatic construction of real numbers interesting?

When you go to class, they usually tell you that that the real numbers and all of its properties and theorems can be constructed from the 11 field axioms.

The axioms are stated, then they usufulness is usually shown by the proof that

$$a \cdot 0 = 0, \forall a \in \mathbb{R}.$$

Wow, you did not expect that one.

Then other marvelouse things come like:

$$-(-a) = a, \forall a \in \mathbb{R}.$$

And the list goes go on and on.

It is exremely tedious to learn those proofs. Basically students learn how to memorize them.

Well, I found my peace with this topic when I started thinking of what happens when some axioms is not used, and what canNOT be proven.

So here goes my example:

Lets just go with the ring axioms. So we don't have the multiplicative identity of the elements ensured, i.e.:

$$a \cdot a^{-1} = 1, \forall a.$$

So what happens, well you can't ensure that a ring is a integral domain. What was that, well basically in an integral domain:

$$a \cdot b = 0 \Leftrightarrow a = 0 \ or \ b = 0.$$

So that means that in sructures that are not integral domains, we have elements that can satisfy that equation without beeing zero. Like in mod 6 we have that:

$$2 \cdot 3 = 6 \equiv 0 \pmod 6$$

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So, I encourage everyone to find out their favorite property of reals that can't be proven with all the fancy axioms.

Go and do some real math. . . .