Primes: the indivisible and the indispensable (2)

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Fundamental Theorem of Arithmetic

“Could there possibly be the largest prime number?”

We’ll set aside the answer to this question for now and explore the etymology of the term "prime number." As mentioned previously, academic discourse on prime numbers began with ancient Greek philosophers, who referred to prime numbers as **πρῶτος(protos)**, meaning "first" or "primary." This term was later translated into the Latin word **primus**, which evolved into the modern English word prime.

In Korea and Japan, prime numbers are referred to as **소수(sosu)** and そすう **(sosū)** respectively, each pronouncing 素數 in their own way. Here, the character "素" means "basic" or "foundational." According to Japanese sources, the term was first coined in 1881 as a translation of "prime number," and it seems that Korea subsequently adopted it. In China, the character "质," with the same meaning, is used instead, calling prime numbers 质数**(zhìshù)**. In North Korea, they are referred to as 씨수, literally meaning "seed numbers." Despite the variety of names, the shared idea is that prime numbers are foundational. Foundational to what, exactly?

When counting objects, we use numbers like 1, 2, 3, and so on, collectively called **Natural numbers**.[1] Natural numbers can be categorized as 1, prime numbers, or composite numbers. By definition, composite numbers have divisors other than 1 and themselves, making them expressible as products of prime numbers. For example, 15 can be factored into 3×5, and 100 can be factored into $2^2 \times 5^2$. This process of expressing natural numbers as products of primes is known as prime factorization.

Every natural number except 1 can be uniquely factorized into primes. The most astonishing aspect of this is the uniqueness. No matter how creatively one approaches the task of factorizing 15, it always reduces to 3×5. Though this is a familiar concept, its significance becomes extraordinary when you ruminate on it: What is it about primes that enables them to uniquely factor all natural numbers? Mathematicians named this wondrous property: the **Fundamental Theorem of Arithmetic**.[2]

Theorem 1 (Fundamental Theorem of Arithmetic). *Every natural number greater than* 1 *can be uniquely factorized into prime numbers.*

If we were to rename prime numbers with a different term, what might be most fitting? I would propose atomos numbers. The English word atom originates from the Greek **ἄτομος(atomos)**, which combines the prefix **ἄ-**, meaning "not," with **τέμνω**(temno), meaning “to cut” or “to divide.” Thus, atomos literally means “indivisible” or “uncuttable,” which aligns perfectly with the essence of a prime number. While we now know that atoms consist of protons, neutrons, and electrons, the original meaning of atom makes the analogy of primes as the “atoms of numbers” very apt.

Just as atoms are the fundamental building blocks of the physical world, prime numbers are the foundational building blocks of the number system. If atomic theory posits that all matter consists of combinations of atoms, the Fundamental Theorem of Arithmetic declares that all natural numbers are composed of combinations of primes. Many properties and mysteries of numbers can be traced back to primes. Furthermore, a remarkable aspect of the mathematical universe is that prime composition is much simpler than atomic composition. While atoms combine in various ways—through metallic, ionic, or covalent bonds—to form matter, primes combine using only multiplication. Additionally, in chemistry, even substances made up of the same atoms can differ based on their arrangement and structure,[3] but this is not the case for primes. Multiplying 3×5 or 5×3 will always yield 15.

This brings us to the reason why 1 is not considered a prime number. If 1 were treated as prime, the uniqueness of prime factorization would be lost. For example, 6 could be factorized as 2×3, $1 \times 2 \times 3$, or $1 \times 1 \times 2 \times 3$. While the definition of primes as numbers divisible only by 1 and themselves might include 1 as a prime, mathematicians intentionally excluded it to preserve the beauty and integrity of the Fundamental Theorem of Arithmetic. This decision has proven so compelling that, while the question of whether 0 should be a natural number remains debated, the consensus that “1 is not a prime” is nearly universal.

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1. Whether 0 is a natural number has been a long-standing debate in mathematics. Generally, Continental Europeans consider 0 a natural number, while Russia and English-speaking countries typically do not.↩

2. It is a convention in mathematics to name the most central theorem in a field the “Fundamental Theorem.” Examples include the Fundamental Theorem of Calculus, the Fundamental Theorem of Linear Algebra, and the Fundamental Theorem of Galois Theory, among others.↩
3. Substances with identical molecular formulas but different structures and properties are called isomers.↩