My definition of infinitieth derivative

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This post is about an early definition I made, which I call infinitieth derivatives, or more precisely ω-th derivative. The ω-th derivative of a smooth function f from $\mathbb{R}$ to $\mathbb{R}$ is the pointwise limit of the sequence of n-th derivatives f, f', f'', etc, assuming, of course, that the pointwise limit does in fact exist for every x in $\mathbb{R}$.

To give some examples, the ω-th derivative of every polynomial is the 0 function. The ω-th derivative of the exponential function e^x is e^x. Slightly more interestingly, the ω-th derivative of the exponential function 2^x is the 0 function. The case of 2^x is interesting because the n-th derivatives of 2^x converge pointwise but not uniformly to the 0 function.

However, it seems that "most" smooth functions do not have a ω-th derivative, like sine or cosine or an exponential function b^x with $b > e$.

The reason this definition is interesting, is because we can take the derivative of the ω-th derivative itself, to get the $\omega + 1$-th derivative, and keep iterating this process thoroughout the transfinite ordinals, where, at limit ordinals, we take the limit of the sequence of derivatives thus far.

However, I have noticed that in every case I have examined so far, the ω-th derivative is a scalar multiple of the exponential function e^x, which forces all further derivatives to be the same as the ω-th derivative, thus trivializing the notion of ordinal derivatives. What I really want to know is whether the ω-th derivative of every real function that has a ω-th derivative is a scalar multiple of the exponential function e^x.