Proof that a relation is an equivalence relation if and only if it is both reflexive and circular

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Like I said in the last post, this post will prove that a binary relation R on a set S is an equivalence relation if and only if it is both reflexive and circular. So, assume R is an equivalence relation. We have to prove that R is both reflexive and circular. Since reflexivity is one of the conditions of being an equivalence relation, we only really have to prove that R is circular. Let x, y, z be elements of S, and assume that $xRy \wedge yRz$. We have to show that zRx. By transitivity, xRz, and then by symmetry, zRx, so we are done. Now, for the other direction, assume R is both reflexive and circular. We have to prove R is an equivalence relation. We already have reflexivity, so we have to prove that R is both symmetric and transitive. I will prove this by first proving symmetry, and then using symmetry to prove transitivity. So, for the proof of symmetry, assume xRy. We have to show that yRx. Since R is reflexive, we know that xRx. By circularity, we have yRx, so we are done. Now, for transitivity, assume $xRy \wedge yRz$. We have to show that xRz. By circularity, zRx, and then by the symmetry that was just proved, xRz, so we are done. The proof is complete.