My definition of circular relations

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This is my first post on this blog. This post is about one of the first mathematical definitions I ever made in my life, which I call circular relations. A circular relation is a binary relation R on a set S, such that the following property holds:

$(\forall x)(\forall y)(\forall z)((xRy \wedge yRz) \rightarrow zRx)$.

Note the contrast with transitivity, where instead of xRz in the consequent, we have zRx. An example of a relation having this property is the “defeats” relation in the game of Rock-Paper-Scissors: Rock defeats Scissors, Scissors defeats Paper, but Paper defeats Rock. My next post will be about proving a theorem about circular relations, namely that a binary relation R is an equivalence relation if and only if R is both a reflexive and circular relation.