P does not imply Q: The difference between implication and conditional

User 2473 • 22 Apr 2026

There is a very common misconception that even a lot of professional mathematicians have, which is the idea that implication is the same thing as conditional. The purpose of this post is to clear up that misconception, and explain the difference between implication and conditional.

The relationship between implication and conditional can be summarized in this short phrase: Implication is tautologyhood of the conditional. Let me explain what I mean.

Consider the system of propositional logic with a countably infinite set of propositional variables $\{p_1, p_2, ..., p_n, ...\}$, a pair of parentheses (and), as well as the logical operators $\neg$, $\vee$, $\wedge$ and $\supset$, which represent negation, disjunction, conjunction, and conditional, respectively.

We define the set of well-formed formulas inductively as follows: first, every propositional variable by itself is a well-formed formula. Next, if A is a formula, then so is $\neg A$. And finally, if A and B are formulas, so is $(A * B)$, where $*$ can represent either $\vee$, $\wedge$, or $\supset$.

A tautology of the propositional logic is defined as follows: First, we define a Boolean valuation as a function v from the set of well-formed formulas to the set $\{0, 1\}$ that respects these properties, for any well-formed formulas A and B:

1.$v(\neg A) = 1 - v(A)$

2.$v((A \vee B)) = min(v(A), v(B))$

3.$v((A \wedge B)) = max(v(A), v(B))$

4.$v((A \supset B)) = min(1 - v(A), v(B))$

A tautology is then defined as a well-formed formula which is mapped to 1 under every Boolean valuation.

Now we come to the definition of implication. Implication is a binary relation on the set of well-formed formula that is defined as follows: for any well-formed formulas A and B, the relation $A \to B$, read "A implies B", holds if and only if the well-formed formula $(A \supset B)$ is a tautology.

Let me give some examples and also some nonexamples. p_1 implies $(p_1 \vee p_2)$, because the well-formed formula $(p_1 \supset (p_1 \vee p_2))$ is a tautology. Also, for any well-formed formula A, A implies A. As a nonexample, p_1 does not imply p_2, because the formula $(p_1 \supset p_2)$ is not a tautology: There is a Boolean valuation that maps $(p_1 \supset p_2)$ to 0. The last point is precisely the reason why I wrote "P does not imply Q" in the title: If we write p_1 as P, and p_2 as Q, then indeed P does not imply Q.

That said, there is still one more subtle point to clear up. In most mathematical contexts, P and Q are variables ranging over arbitrary propositions. In those contexts, whether P implies Q depends on what the content of P and Q are. I used A and B as variables that range over arbitrary well-formed formulas, but of course one can use P and Q instead. So, basically, under my convention, P does not imply Q, but the question of whether the formula A implies the formula B is indeterminate until we know what A and B are. Some other mathematician could have the opposite convention, and use A and B as two distinct propositional variables, while using P and Q as variables ranging over arbitrary formulas. So perhaps I should have written "p_1 does not imply p_2" in the title. But in any case, I hope that now I have clearly explained the distinction between implication and conditional.