Two rules, same function; two functions, same rule

written by User 2473 on Functor Network
original link: https://functor.network/user/2473/entry/1055

This post is about an interesting observation I made regarding rules and functions. One function can have two distinct rules. For example, consider the rules x^3 and x. They are different, yet over the set $\{-1, 0, 1\}$, they define the same function. And the opposite can also happen. For example, the function x^2 over the reals $\mathbb{R}$ is different than the function x^2 over the rationals $\mathbb{Q}$, but they have the same rule, namely x^2.

Of course, to make this all precise, we would need a rigorous definition of "rule" that can distinguish it from functions. I invite any readers to try to come up with a rigorous definition of "rule".