

Every open cover has a countable subcover \Rightarrow a countable dense set

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Let X be a metric space.

Every open cover in X has a countable subcover \Rightarrow a countable dense set
 $\{B(1/n, x) \mid x \text{ in } X\}$ is clearly an open cover, whatever $n > 0$ one picks. for a
fixed n , say $n=2$, there is a subcover $\{B(1/n, x_m) \mid x_m, m = 1, 2, 3, \dots\}$ by the
assumption. To get a dense set, we just include x_m for all m and n .

The converse also holds.