Every open cover has a countable subcover => a countable dense set

written by User 2237 on Functor Network
original link: https://functor.network/user/2237/entry/830

Let X be a metric space.
Every open cover in X has a countable subcover => a countable dense set
{B(1/n, x) | all x in X} is clearly an open cover, whatever n>0 one picks. for a fixed n, say n=2, there is a subcover $\{\mathtt{B(1/n, x_{mn})|x_{mn}, m = 1, 2, 3, ...}\}$ by the assumption. To get a dense set, we just include xmn for all m and n.

The converse also holds.