Every open cover has a countable subcover => a countable dense set

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Let X be a metric space.
Every open cover in X has a countable subcover => a countable dense set
{B(1/n, x) | all x in X} is clearly an open cover, whatever n>0 one picks. for a fixed n, say n=2, there is a subcover $\{B(1/n, x_{mn}) | x_{mn}, m = 1, 2, 3, ...\}$ by the assumption. To get a dense set, we just include xmn for all m and n.

The converse also holds.