The Correspondence Principle and Computing the Rydberg Constant

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A heuristic guiding principle attributed to Bohr asserts that a quantum theory must approach the corresponding classical theory, which is usually well known to explain most (if not all) of the macroscopic phenomena over many thorough experiments, in the limit of large quantum numbers. That is, classical theories are *macroscopically* correct in the limit where the quantum discontinuities can be considered as infinitesimally small and thus negligible. This principle is called the *correspondence principle*, and we shall attempt to derive a numerical expression for the Rydberg constant using it below.

The Rydberg constant R appears in Bohr's expression for the energy of an electron orbiting a nucleus in the n-th energy level

$$E_n = -\frac{Rh}{n^2}$$

We will assert without showing, due to classical electromagnetic theory and Keppler's law, that the frequency of the radiation emitted by such an electron as above is given by

$$\nu_{\text{el}} = \frac{1}{\pi e^2}\left(\frac{2\left|E\right|^3}{m}\right)$$

which is emitted continuously as the electron loses energy. The same loss of energy occurs in Bohr's theory through discrete jumps.

$$\Delta E_n = \frac{\mathrm{d}E_n}{\mathrm{d}n}\,\Delta n = h\frac{2R}{n^3}\Delta n$$

Provided n is large, using the relation $E = h\nu$, the emitted frequency must be a harmonic of a fundamental frequency

$$\nu_0 = \frac{2R}{n^3}$$

We look back at Bohr's energy expression again.

$$
\begin{aligned}
|E_n| &= \frac{Rh}{n^2} \\
\implies \frac{1}{n^3} &= \left(\frac{|E_n|}{Rh}\right)^{\frac{3}{2}} \\
\implies \frac{2R}{n^3} &= 2\left(\frac{|E_n|^3}{Rh^3}\right)^{\frac{1}{2}} \equiv \nu_0
\end{aligned}
$$

Now the *correspondence principle* says that this must approach the classical expression in the limit $n \to \infty$ (whence $E_n \to E$). Equating the two expressions thus

$$
2\left(\frac{|E|^3}{Rh^3}\right)^{\frac{1}{2}} = \frac{1}{\pi e^2}\left(\frac{2\,|E|^3}{m}\right)^{\frac{1}{2}}
$$

yields the following expression for R

$$
R = \frac{2m\pi^2 e^4}{h^3}
$$

which gives a value remarkably close to the presently known experimental value.

The above article is my own recollection of the section on the application of the correspondence principle in the calculation of the Rydberg constant in (Messiah 2014).

Reference

Messiah, Albert. 2014. *Quantum Mechanics*. Dover Publications.