# Brennan 1998: The Role of Learning in Dynamic Portfolio Decisions

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2 assets: bond and stock

1. bond gives constant rate of return $r$
2. stock price: $\frac{dS}{S} = \mu dt + \sigma dz$

wealth process:

$\frac{dW}{W} = [r + \alpha(\mu - r)]dt + \alpha\sigma dz$ $\alpha$ here is fraction of wealth in risky asset

Investor unable to observe $\mu$. At $t = 0$, investor views the distro of $\mu$ as $N(m_0, v_0)$.

We denote the conditional expectation and variance of $\mu$ by $m_t, v_t$. By Lipster-Shiryayev 1978 (see Gennotte 1986 for a simple exposition), we have:

1. $dm = \frac{v_t}{\sigma^2}[\frac{dS}{S} - mdt]$
2. $dv = [-\frac{v^2}{\sigma^2}]dt$ (deterministic dynamics)

investor's value function at $t$ depends on current wealth, current assessment of $\mu$, and $t$. Denote it as $J(W, m, t)$.

formulate HJB, then use ansatz $J(W, m, t) = \frac{1}{\gamma}W^{\gamma}u(m, t)$, and derive an HJB for $u(m, t)$.