General topology (Autumn 2024), 1

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As of today, I'm taking over teaching general topology for an ill colleague. I've taught a course like this a few times before, both at Harvard and here at Edinburgh (but never at MIT, somehow). In 2020, I even put some topology lectures online (some of which are kind of hokey, but you can't expect much from a guy who's been stuck it his house for weeks). [1]

Anyhow, general topology is a class I really enjoy, and I've been reflecting why.

1. One reason is that there are so many different ways to tell the story. You can define a topology as a system of opens, or a system of closeds, or as a closure operator. Each of those tells you something about what topologies capture. It's just fun to explore the different narratives.

2. General topology is usually the first time students encounter real *beasts* in mathematics. The definition of a topological space is so extremely general that you need to restrain the concept with a whole bunch of conditions to keep wacky "pathological" topologies out.

3. But then a lot of those beasts turn out to be friends. The Cantor space seems weird and unnecessary, like a trick you use to annoy students. Then you learn about the p-adic integers. Eventually the Cantor space just starts to feel like what you have to do sometimes to pass from finite discrete structures to infinite ones.

4. More generally, you start to appreciate the following idea: when you have an algebraic structure that isn't finite or finitely generated, your only hope of dealing with it responsibly is to incorporate a topology. You see this with the p-adics, but you also see it with absolute Galois groups, Lie groups, topological vector spaces, infinite-dimensional representations, etc., etc.

Topology resembles a viscous liquid poured over the rough surface of students' understanding. Slowly the idea seeps into all sorts of nooks and crannies and gaps in students' mathematical worldview. The rough surface over time becomes much smoother and much more navigable. It's a powerful thing, but appreciation for it develops only gradually.

But as a lecturer I have to *start* somewhere, and first lectures freak me out. The students and I are strangers (usually), and yet I'm implicitly asking them for the most precious resource they have – their attention. 50 minutes is a lot of their time, but it's not nearly enough of mine. I have to work out what they already know, what they don't yet know, what they think is important, what they think is a waste of time, what they think is interesting, what they think is boring, etc., etc. – and I *have to be prepared to try to change their minds about some of this*. None of this includes actually communicating the mathematics itself correctly!

It doesn't help that students in the UK generally expect more organization and more structure than I can deliver. I think lecturers here are usually much more orderly and formal. Regimented, even. But long ago, I gave up on the idea that I could ever give lectures that are both thorough and comprehensible to most of my audience. Instead, I come in with a few points I want to underscore, but my goal is a *conversation* with the students. I gently steer that conversation, but I don't press an agenda too hard. Students have questions, confusions, concerns – those are generally my first priority. It's a high-risk, high-reward approach to teaching: on a good day, my lectures are dynamic and interactive; on a bad day, they're chaotic and annoying.

Naturally I'm incredibly nervous beforehand. Just for fun, I looked at my heart rate in the moments just before class started. (I have one of those watches that records that.) It was about the same as when I've been cycling up a gentle hill for five minutes.

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1. And I'm never going to make online math videos as pure and as brilliant as those of Richard Borcherds.↩