Exercises on tensor product

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$\mathbb{Z}/(m) \otimes_{\mathbb{Z}} \mathbb{Z}/(n) = \mathbb{Z}/((m,n))$.

Proof 1.$\otimes$: $([a]_m, [b]_n) \to [ab]_{(m,n)}$ is a well defined linear map from $\mathbb{Z}/(m) \times \mathbb{Z}/(n)$ to $\mathbb{Z}/((m,n))$.

2.For any bilinear operator $\phi\colon \mathbb{Z}/(m) \times \mathbb{Z}/(n) \to M$, $\tilde{\phi}([1]_{(m,n)}) = \phi([1]_m, [1]_n)$ defines a linear operator from $\mathbb{Z}/((m,n))$ to M. This is because $(m,n)\phi([1]_m, [1]_n) = (mx+ny)\phi([1]_m, [1]_n) = \phi([mx]_m, [1]_n) + \phi([1]_m, [ny]_n) = \phi([0]_m, [1]_n) + \phi([1]_m, [0]_n) = 0$, where $mx + ny = (m,n)$.