# Exercises on tensor product

Chun Ding • 26 Dec 2023

> $\mathbb{Z}/(m) \otimes_{\mathbb{Z}} \mathbb{Z}/(n) = \mathbb{Z}/((m,n))$.

**Proof** 1.$\otimes$: $([a]_m, [b]_n) \to [ab]_{(m,n)}$ is a well defined linear map from $\mathbb{Z}/(m) \times \mathbb{Z}/(n)$ to $\mathbb{Z}/((m,n))$.

2.For any bilinear operator $\phi\colon \mathbb{Z}/(m) \times \mathbb{Z}/(n) \to M$, $\tilde{\phi}([1]_{(m,n)}) = \phi([1]_m, [1]_n)$ defines a linear operator from $\mathbb{Z}/((m,n))$ to $M$. This is because $(m,n)\phi([1]_m, [1]_n) = (mx + ny)\phi([1]_m, [1]_n) = \phi([mx]_m, [1]_n) + \phi([1]_m, [ny]_n) = \phi([0]_m, [1]_n) + \phi([1]_m, [0]_n) = 0$,

where $mx + ny = (m,n)$. $\square$