$L^\infty(X, \mu)$ **may fail to be a von Neumann algebra**

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After discussing here, we finally notice that $L^\infty(X, \mu)$ may fail to be a von Neumann algebra. Here is a counter example.

Let X be an uncountable set, μ be a counting measure, and Σ be the family of set A such that A or the complement of A is countable. Let S be a subset of X such that both itself and its complement are uncountable, then the bounded subset $\{\chi_A | A$ is a countable subset of $S\}$ in $L^\infty(X, \mu)$ has no least upper bound.